

# Package ‘fungible’

November 16, 2018

**Version** 1.75

**Date** 2018-11-16

**Title** Psychometric Functions from the Waller Lab

**Author** c(  
 person("Niels", "Waller", role = c("aut", "cre"), email="nwaller@umn.edu" ),  
 person("Jeff", "Jones", role = "ctb"))

**Maintainer** Niels G. Waller <nwaller@umn.edu>

**Depends** R (>= 3.0)

**Imports** clue, e1071, GPArotation, lattice, MASS, methods, mvtnorm,  
 nlqslv, psych, Rcsdp, RSpectra, stringr

**Description** Computes fungible coefficients and Monte Carlo data.

Underlying theory for these functions is described in the following publications:

Waller, N. (2008). Fungible Weights in Multiple Regression. *Psychometrika*, 73(4), 691-703, <DOI10.1007/s11336-008-9066-z>.

Waller, N. & Jones, J. (2009). Locating the Extrema of Fungible Regression Weights. *Psychometrika*, 74(4), 589-602, <DOI10.1007/s11336-008-9087-7>.

Waller, N. G. (2016). Fungible Correlation Matrices:  
A Method for Generating Nonsingular, Singular, and Improper Correlation Matrices for  
Monte Carlo Research. *Multivariate Behavioral Research*, 51(4), 554-568, <DOI10.1080/00273171.2016.1178566>.

Jones, J. A. & Waller, N. G. (2015). The normal-theory and asymptotic distribution-free (ADF)  
covariance matrix of standardized regression coefficients: theoretical extensions  
and finite sample behavior. *Psychometrika*, 80, 365-378, <DOI10.1007/s11336-013-9380-y>.

Waller, N. G. (2018). Direct Schmid-Leiman transformations and rank-  
deficient loadings matrices. *Psychometrika*, 83, 858-870. <DOI 10.1007/s11336-017-9599-0>.

**License** GPL (>=2)

**NeedsCompilation** no

## R topics documented:

adfCor . . . . .	3
adfCov . . . . .	4
BadRBY . . . . .	5
BadRJN . . . . .	6
BadRKtB . . . . .	6
BadRLG . . . . .	7
BadRRM . . . . .	7

BiFAD	8
bigen	10
corSample	12
corSmooth	13
d2r	14
eap	14
eigGen	16
enhancement	17
erf	18
faAlign	19
fals	22
faMAP	23
faSort	25
FMP	26
FMPMonotonicityCheck	29
fungible	30
fungibleExtrema	31
fungibleL	33
fungibleR	35
FUP	39
gen4PMDData	42
genCorr	43
genFMPData	44
HS9Var	46
irf	47
itemDescriptives	48
kurt	50
monte	51
monte1	58
normalCor	59
normF	60
plot.monte	61
r2d	61
rarc	62
rcone	63
rcor	64
rellipsoid	65
restScore	66
rGivens	68
rMAP	69
rmsd	70
RnpdMAP	71
seBeta	74
seBetaCor	75
seBetaFixed	77
skew	78
smoothAPA	79
smoothBY	81
smoothKB	83
smoothLG	84
summary.monte	85
summary.monte1	87

<i>adfCor</i>	3
svdNorm . . . . .	88
tetcor . . . . .	89
tetcorQuasi . . . . .	90
vcos . . . . .	91
vnorm . . . . .	92
<b>Index</b>	<b>93</b>

adfCor	<i>Asymptotic Distribution-Free Covariance Matrix of Correlations</i>
--------	---

**Description**

Function for computing an asymptotic distribution-free covariance matrix of correlations.

**Usage**

adfCor(X, y = NULL)

**Arguments**

- X Data matrix.
- y Optional vector of criterion scores.

**Value**

adfCorMat Asymptotic distribution-free estimate of the covariance matrix of correlations.

**Author(s)**

Jeff Jones and Niels Waller

**References**

Browne, M. W. (1984). Asymptotically distribution-free methods for the analysis of covariance structures. *British Journal of Mathematical and Statistical Psychology*, 37, 62–83.

Steiger, J. H. and Hakstian, A. R. (1982). The asymptotic distribution of elements of a correlation matrix: Theory and application. *British Journal of Mathematical and Statistical Psychology*, 35, 208–215.

**Examples**

```
## Generate non-normal data using monte1
set.seed(123)
## we will simulate data for 1000 subjects
N <- 1000

## R = the desired population correlation matrix among predictors
R <- matrix(c(1, .5, .5, 1), 2, 2)

## Consider a regression model with coefficient of determination (Rsqr):
Rsqr <- .50
```

```
## and vector of standardized regression coefficients
Beta <- sqrt(Rsq/t(sqrt(c(.5, .5))) %*% R %*% sqrt(c(.5, .5))) * sqrt(c(.5, .5))

## generate non-normal data for the predictors (X)
## x1 has expected skew = 1 and kurtosis = 3
## x2 has expected skew = 2 and kurtosis = 5
X <- monte1(seed = 123, nvar = 2, nsub = N, cormat = R, skewvec = c(1, 2),
            kurtvec = c(3, 5))$data

## generate criterion scores
y <- X %*% Beta + sqrt(1-Rsq)*rnorm(N)

## Create ADF Covariance Matrix of Correlations
adfCor(X, y)

#>           12           13           23
#> 12 0.0012078454 0.0005331086 0.0004821594
#> 13 0.0005331086 0.0004980130 0.0002712080
#> 23 0.0004821594 0.0002712080 0.0005415301
```

---

adfCov

---

*Asymptotic Distribution-Free Covariance Matrix of Covariances*


---

## Description

Function for computing an asymptotic distribution-free covariance matrix of covariances.

## Usage

```
adfCov(X, y = NULL)
```

## Arguments

X	Data matrix.
y	Optional vector of criterion scores.

## Value

adfCovMat	Asymptotic distribution-free estimate of the covariance matrix of covariances
-----------	---

## Author(s)

Jeff Jones and Niels Waller

## References

Browne, M. W. (1984). Asymptotically distribution-free methods for the analysis of covariance structures. *British Journal of Mathematical and Statistical Psychology*, 37, 62–83.

**Examples**

```

## Generate non-normal data using monte1
set.seed(123)

## we will simulate data for 1000 subjects
N <- 1000

## R = the desired population correlation matrix among predictors
R <- matrix(c(1, .5, .5, 1), 2, 2)

## Consider a regression model with coefficient of determination (Rsqr):
Rsqr <- .50

## and vector of standardized regression coefficients
Beta <- sqrt(Rsqr/t(sqrt(c(.5, .5)))) %*% R %*% sqrt(c(.5, .5)) * sqrt(c(.5, .5))

## generate non-normal data for the predictors (X)
## x1 has expected skew = 1 and kurtosis = 3
## x2 has expected skew = 2 and kurtosis = 5
X <- monte1(seed = 123, nvar = 2, nsub = N, cormat = R, skewvec = c(1, 2),
            kurtvec = c(3, 5))$data

## generate criterion scores
y <- X %*% Beta + sqrt(1-Rsqr)*rnorm(N)

## Create ADF Covariance Matrix of Covariances
adfCov(X, y)

#>      11      12      13      22      23      33
#> 11 3.438760 2.317159 2.269080 2.442003 1.962584 1.688631
#> 12 2.317159 3.171722 2.278212 3.349173 2.692097 2.028701
#> 13 2.269080 2.278212 2.303659 2.395033 2.149316 2.106310
#> 22 2.442003 3.349173 2.395033 6.275088 4.086652 2.687647
#> 23 1.962584 2.692097 2.149316 4.086652 3.287088 2.501094
#> 33 1.688631 2.028701 2.106310 2.687647 2.501094 2.818664

```

---

BadRBY

---

*Improper correlation matrix reported by Bentler and Yuan*


---

**Description**

Example improper R matrix reported by Bentler and Yuan (2011)

**Usage**

```
data(BadRBY)
```

**Format**

A 12 by 12 non-positive definite correlation matrix.

**Source**

Bentler, P. M. & Yuan, K. H. (2011). Positive definiteness via off-diagonal scaling of a symmetric indefinite matrix. *Psychometrika*, 76(1), 119–123.

**Examples**

```
data(BadRBY)
```

---

BadRJN

*Improper R matrix reported by Joseph and Newman*

---

**Description**

Example NPD improper correlation matrix reported by Joseph and Newman

**Usage**

```
data(BadRJN)
```

**Format**

A 14 by 14 non-positive definite correlation matrix.

**Source**

Joseph, D. L. & Newman, D. A. (2010). Emotional intelligence: an integrative meta-analysis and cascading model. *Journal of Applied Psychology*, 95(1), 54–78.

**Examples**

```
data(BadRJN)
```

---

BadRKtB

*Improper R matrix reported by Knol and ten Berge*

---

**Description**

Example improper R matrix reported by Knol and ten Berge

**Usage**

```
data(BadRKtB)
```

**Format**

A 6 by 6 non-positive definite correlation matrix.

**Source**

Knol, D. L. and Ten Berge, J. M. F. (1989). Least-squares approximation of an improper correlation matrix by a proper one. *Psychometrika*, 54(1), 53-61.

**Examples**

```
data(BadRKtB)
```

---

BadRLG	<i>Improper R matrix reported by Lurie and Goldberg</i>
--------	---

---

**Description**

Example improper R matrix reported by Lurie and Goldberg

**Usage**

```
data(BadRLG)
```

**Format**

A 3 by 3 non-positive definite correlation matrix.

**Source**

Lurie, P. M. & Goldberg, M. S. (1998). An approximate method for sampling correlated random variables from partially-specified distributions. *Management Science*, 44(2), 203–218.

**Examples**

```
data(BadRLG)
```

---

BadRRM	<i>Improper R matrix reported by Rousseeuw and Molenberghs</i>
--------	--

---

**Description**

Example improper R matrix reported by Rousseeuw and Molenberghs

**Usage**

```
data(BadRRM)
```

**Format**

A 3 by 3 non-positive definite correlation matrix.

**Source**

Rousseeuw, P. J. & Molenberghs, G. (1993). Transformation of non positive semidefinite correlation matrices. *Communications in Statistics–Theory and Methods*, 22(4), 965–984.

**Examples**

```
data(BadRRM)
```

---

BiFAD

*Bifactor Analysis via Direct Schmid Leiman Transformations*


---

**Description**

Bifactor Analysis via Direct Schmid Leiman Transformations

**Usage**

```
BiFAD(R, B = NULL, nGroup = NULL,
      factorMethod = "minres",
      rotation="oblimin", salient = .25,
      maxitFA = 5000,
      maxitRotate = 5000,
      gamma = 0)
```

**Arguments**

R	Input correlation matrix.
B	Bifactor target matrix. If B=NULL the program will create an empirically defined target matrix.
nGroup	Number of group factors in bifactor solution.
factorMethod	Factor extraction method. Options include: minres (minimum residual), ml (maximum likelihood), pa (principal axis), gls (generalized least squares).
rotation	Factor rotation method. Current options include: oblimin, geominQ, quartimin, promax.
salient	Threshold value for creating an empirical target matrix.
maxitFA	Maximum iterations for the factor extraction method.
maxitRotate	Maximum iterations for the gradient pursuit rotation algorithm.
gamma	Optional tuning parameter for oblimin rotation.

**Value**

B	User defined or empirically generated target matrix.
BstarSL	Direct S-L solution.
BstarFR	Direct full rank bifactor solution.
rmsrSL	Root mean squared residual of (B - BstarSL).
rmsrFR	Root mean squared residual of (B - BstarFR).



**Author(s)**

Niels Waller

**References**

Waller, N. G. (2018). Direct Schmid Leiman transformations and rank deficient loadings matrices. *Psychometrika*, 83, 858-870.

**Examples**

```
cat("\nExample 1:\nEmpirical Target Matrix:\n")
# Mansolf and Reise Table 2 Example
Btrue <- matrix(c(.48, .40, 0, 0, 0,
                  .51, .35, 0, 0, 0,
                  .67, .62, 0, 0, 0,
                  .34, .55, 0, 0, 0,
                  .44, 0, .45, 0, 0,
                  .40, 0, .48, 0, 0,
                  .32, 0, .70, 0, 0,
                  .45, 0, .54, 0, 0,
                  .55, 0, 0, .43, 0,
                  .33, 0, 0, .33, 0,
                  .52, 0, 0, .51, 0,
                  .35, 0, 0, .69, 0,
                  .32, 0, 0, 0, .65,
                  .66, 0, 0, 0, .51,
                  .68, 0, 0, 0, .39,
                  .32, 0, 0, 0, .56), 16, 5, byrow=TRUE)

Rex1 <- Btrue %*% t(Btrue)
diag(Rex1) <- 1

out.ex1 <- BiFAD(R = Rex1, B = NULL, nGroup = 4,
                 factorMethod = "minres",
                 rotation="oblimin", salient = .25,
                 maxitFA = 5000,
                 maxitRotate = 5000,
                 gamma = 0)

cat("\nRank Deficient Bifactor Solution:\n")
print( round(out.ex1$BstarSL, 2) )

cat("\nFull Rank Bifactor Solution:\n")
print( round(out.ex1$BstarFR, 2) )

cat("\nExample 2:\nUser Defined Target Matrix:\n")

Bpattern <- matrix(c( 1, 1, 0, 0, 0,
                      1, 1, 0, 0, 0,
                      1, 1, 0, 0, 0,
                      1, 1, 0, 0, 0,
                      1, 0, 1, 0, 0,
                      1, 0, 1, 0, 0,
                      1, 0, 1, 0, 0,
                      1, 0, 1, 0, 0,
                      1, 0, 0, 1, 0,
```

```

      1,  0,  0,  1,  0,
      1,  0,  0,  1,  0,
      1,  0,  0,  1,  0,
      1,  0,  0,  0,  1,
      1,  0,  0,  0,  1,
      1,  0,  0,  0,  1,
      1,  0,  0,  0,  1,
      1,  0,  0,  0,  1), 16, 5, byrow=TRUE)

out.ex2 <- BiFAD(R = Rex1, B = Bpattern, nGroup = NULL,
  factorMethod = "minres",
  rotation="oblimin", salient = .25,
  maxitFA = 5000,
  maxitRotate = 5000,
  gamma = 0)

cat("\nRank Deficient Bifactor Solution:\n")
print( round(out.ex2$BstarSL, 2) )

cat("\nFull Rank Bifactor Solution:\n")
print( round(out.ex2$BstarFR, 2) )

```

bigen

*Generate Correlated Binary Data***Description**

Function for generating binary data with population thresholds.

**Usage**

```
bigen(data, n, thresholds = NULL, Smooth = FALSE, seed = NULL)
```

**Arguments**

data	Either a matrix of binary (0/1) indicators or a correlation matrix.
n	The desired sample size of the simulated data.
thresholds	If data is a correlation matrix, thresholds must be a vector of threshold cut points.
Smooth	(logical) Smooth = TRUE will smooth the tetrachoric correlation matrix.
seed	Default = FALSE. Optional seed for random number generator.

**Value**

data	Simulated binary data
r	Input or calculated (tetrachoric) correlation matrix

**Author(s)**

Niels G Waller

**Examples**

```

## Example: generating binary data to match
## an existing binary data matrix
##
## Generate correlated scores using factor
## analysis model
##  $X \leftarrow Z * L' + U * D$ 
## Z is a vector of factor scores
## L is a factor loading matrix
## U is a matrix of unique factor scores
## D is a scaling matrix for U

N <- 5000

# Generate data from a single factor model
# factor patter matrix
L <- matrix( rep(.707, 5), nrow = 5, ncol = 1)

# common factor scores
Z <- as.matrix(rnorm(N))

# unique factor scores
U <- matrix(rnorm(N * 5), nrow = N, ncol = 5)
D <- diag(as.vector(sqrt(1 - L^2)))

# observed scores
X <- Z %*% t(L) + U %*% D

cat("\nCorrelation of continuous scores\n")
print(round(cor(X),3))

# desired difficulties (i.e., means) of
# the dichotomized scores
difficulties <- c(.2, .3, .4, .5, .6)

# cut the observed scores at these thresholds
# to approximate the above difficulties
thresholds <- qnorm(difficulties)

Binary <- matrix(0, N, ncol(X))
for(i in 1:ncol(X)){
  Binary[X[,i] <= thresholds[i],i] <- 1
}

cat("\nCorrelation of Binary scores\n")
print(round(cor(Binary), 3))

## Now use 'bigen' to generate binary data matrix with
## same correlations as in Binary

z <- bigen(data = Binary, n = N)

cat("\n\nnames in returned object\n")
print(names(z))

cat("\nCorrelation of Simulated binary scores\n")

```

```
print(round(cor(z$data), 3))

cat("Observed thresholds of simulated data:\n")
cat(apply(z$data, 2, mean))
```

corSample

*Sample Correlation Matrices from a Population Correlation Matrix***Description**

Sample correlation (covariance) matrices from a population correlation matrix (see Browne, 1968; Kshirsagar, 1959)

**Usage**

```
corSample(R, n)
```

**Arguments**

R	A population correlation matrix.
n	Sample correlation (covariance) matrices will be generated assuming a sample size of n.

**Value**

cor.sample	Sample correlation matrix.
cov.sample	Sample covariance matrix.

**Author(s)**

Niels Waller

**References**

Browne, M. (1968). A comparison of factor analytic techniques. *Psychometrika*, 33(3), 267-334.

Kshirsagar, A. (1959). Bartlett decomposition and Wishart distribution. *The Annals of Mathematical Statistics*, 30(1), 239-241.

**Examples**

```
R <- matrix(c(1, .5, .5, 1), 2, 2)
# generate a sample correlation from pop R with n = 25
out <- corSample(R, n = 25)
out$cor.sample
out$cov.sample
```

corSmooth

*Smooth a Non PD Correlation Matrix***Description**

A function for smoothing a non-positive definite correlation matrix by the method of Knol and Berger (1991).

**Usage**

```
corSmooth(R, eps = 1E8 * .Machine$double.eps)
```

**Arguments**

R	A non-positive definite correlation matrix.
eps	Small positive number to control the size of the non-scaled smallest eigenvalue of the smoothed R matrix. Default = $1E8 * .Machine$double.eps$

**Value**

Rsmoothed	A Smoothed (positive definite) correlation matrix.
-----------	--

**Author(s)**

Niels Waller

**References**

Knol, D. L., and Berger, M. P. F., (1991). Empirical comparison between factor analysis and multi-dimensional item response models. *Multivariate Behavioral Research*, 26, 457-477.

**Examples**

```
## choose eigenvalues such that R is NPD
l <- c(3.0749126, 0.9328397, 0.5523868, 0.4408609, -0.0010000)

## Generate NPD R
R <- genCorr(eigenval = l, seed = 123)
print(eigen(R)$values)

#> [1] 3.0749126 0.9328397 0.5523868 0.4408609 -0.0010000

## Smooth R
Rsm<-corSmooth(R, eps = 1E8 * .Machine$double.eps)
print(eigen(Rsm)$values)

#> [1] 3.074184e+00 9.326669e-01 5.523345e-01 4.408146e-01 2.219607e-08
```

---

d2r	<i>Convert Degrees to Radians</i>
-----	-----------------------------------

---

**Description**

A simple function to convert degrees to radians

**Usage**

```
d2r(deg)
```

**Arguments**

deg	Angle in degrees.
-----	-------------------

**Value**

Angle in radians.

**Examples**

```
d2r(90)
```

---

eap	<i>Compute eap trait estimates for FMP and FUP models</i>
-----	---

---

**Description**

Compute eap trait estimates for items fit by filtered monotonic polynomial IRT models.

**Usage**

```
eap(data, bParams, NQuad = 21, priorVar = 2, mintheta = -4, maxtheta = 4)
```

**Arguments**

data	N(subjects)-by-p(items) matrix of 0/1 item response data.
bParams	A p-by-9 matrix of FMP or FUP item parameters and model designations. Columns 1 - 8 hold the (possibly zero valued) polynomial coefficients; column 9 holds the value of k.
NQuad	Number of quadrature points used to calculate the eap estimates.
priorVar	Variance of the normal prior for the eap estimates. The prior mean equals 0.
mintheta, maxtheta	NQuad quadrature points will be evenly spaced between mintheta and maxtheta

**Value**

eap trait estimates.

**Author(s)**

Niels Waller

**Examples**

```
## this example demonstrates how to calculate
## eap trait estimates for a scale composed of items
## that have been fit to FMP models of different
## degree

NSubjects <- 2000

## Assume that
## items 1 - 5 fit a k=0 model,
## items 6 - 10 fit a k=1 model, and
## items 11 - 15 fit a k=2 model.

itmParameters <- matrix(c(
  # b0    b1    b2    b3    b4  b5, b6, b7, k
  -1.05, 1.63,  0.00, 0.00, 0.00, 0,    0, 0, 0, #1
  -1.97, 1.75,  0.00, 0.00, 0.00, 0,    0, 0, 0, #2
  -1.77, 1.82,  0.00, 0.00, 0.00, 0,    0, 0, 0, #3
  -4.76, 2.67,  0.00, 0.00, 0.00, 0,    0, 0, 0, #4
  -2.15, 1.93,  0.00, 0.00, 0.00, 0,    0, 0, 0, #5
  -1.25, 1.17, -0.25, 0.12, 0.00, 0,    0, 0, 1, #6
  1.65, 0.01,  0.02, 0.03, 0.00, 0,    0, 0, 1, #7
  -2.99, 1.64,  0.17, 0.03, 0.00, 0,    0, 0, 1, #8
  -3.22, 2.40, -0.12, 0.10, 0.00, 0,    0, 0, 1, #9
  -0.75, 1.09, -0.39, 0.31, 0.00, 0,    0, 0, 1, #10
  -1.21, 9.07,  1.20,-0.01,-0.01, 0.01, 0, 0, 2, #11
  -1.92, 1.55, -0.17, 0.50,-0.01, 0.01, 0, 0, 2, #12
  -1.76, 1.29, -0.13, 1.60,-0.01, 0.01, 0, 0, 2, #13
  -2.32, 1.40,  0.55, 0.05,-0.01, 0.01, 0, 0, 2, #14
  -1.24, 2.48, -0.65, 0.60,-0.01, 0.01, 0, 0, 2),#15
  15, 9, byrow=TRUE)

# generate data using the above item parameters
ex1.data<-genFMPData(NSubj = NSubjects, bParams = itmParameters,
  seed = 345)$data

## calculate eap estimates for mixed models
thetaEAP<-eap(data = ex1.data, bParams = itmParameters,
  NQuad = 25, priorVar = 2,
  mintheta = -4, maxtheta = 4)

## compare eap estimates with initial theta surrogates

if(FALSE){  #set to TRUE to see plot

  thetaInit <- svdNorm(ex1.data)
  plot(thetaInit,thetaEAP, xlim = c(-3.5,3.5),
    ylim = c(-3.5,3.5),
    xlab = "Initial theta surrogates",
```





```

    type = "b",
    main = plotTitle,
    ylim = c(0, maxy),
    xlab = "Dimensions",
    ylab = "Eigenvalues",
    cex.main = .9)

```

enhancement

*Find OLS Regression Coefficients that Exhibit Enhancement***Description**

Find OLS regression coefficients that exhibit a specified degree of enhancement.

**Usage**

```
enhancement(R, br, rr)
```

**Arguments**

R	Predictor correlation matrix.
br	Model R-squared = $b' r$ . That is, br is the model coefficient of determination: $b' R b = R_{sq} = br$
rr	Sum of squared predictor-criterion correlations ( $rx y$ ). That is, $rr = r' r = \text{Sum}(rx y^2)$

**Value**

b	Vector of standardized regression coefficients.
r	Vector of predictor-criterion correlations.

**Author(s)**

Niels Waller

**References**

Waller, N. G. (2011). The geometry of enhancement in multiple regression. *Psychometrika*, 76, 634–649.

**Examples**

```

## Example: For a given predictor correlation matrix (R) generate
## regression coefficient vectors that produce enhancement (br - rr > 0)

## Predictor correlation matrix
R <- matrix(c( 1, .5, .25,
               .5, 1, .30,
               .25, .30, 1), 3, 3)

## Model coefficient of determination

```

```

Rsqr <- .60

output<-enhancement(R, br = Rsqr, rr =.40)

r <- output$r
b <- output$b

##Standardized regression coefficients
print(t(b))

##Predictor-criterion correlations
print(t(r))

##Coefficient of determinations (b'r)
print(t(b) %*% r)

##Sum of squared correlations (r'r)
print(t(r) %*% r)

```

---

erf	<i>Utility fnc to compute the components for an empirical response function</i>
-----	---

---

## Description

Utility function to compute empirical response functions.

## Usage

```
erf(theta, data, whichItem, min = -3, max = 3, Ncuts = 12)
```

## Arguments

theta	Vector of estimated latent trait scores.
data	A matrix of binary item responses.
whichItem	Data for an erf will be generated for whichItem.
min	Default = -3. Minimum value of theta.
max	Default = 3. Maximum value of theta.
Ncuts	Number of score groups for erf.

## Value

probs	A vector (of length Ncuts) of bin response probabilities for the empirical response function.
centers	A vector of bin centers.
Ni	Bin sample sizes.
se.p	Standard errors of the estimated bin response probabilities.

## Author(s)

Niels Waller

## Examples

```

NSubj <- 2000

#generate sample k=1 FMP data
b <- matrix(c(
  #b0    b1    b2    b3    b4    b5 b6 b7 k
  1.675, 1.974, -0.068, 0.053, 0, 0, 0, 0, 1,
  1.550, 1.805, -0.230, 0.032, 0, 0, 0, 0, 1,
  1.282, 1.063, -0.103, 0.003, 0, 0, 0, 0, 1,
  0.704, 1.376, -0.107, 0.040, 0, 0, 0, 0, 1,
  1.417, 1.413, 0.021, 0.000, 0, 0, 0, 0, 1,
  -0.008, 1.349, -0.195, 0.144, 0, 0, 0, 0, 1,
  0.512, 1.538, -0.089, 0.082, 0, 0, 0, 0, 1,
  0.122, 0.601, -0.082, 0.119, 0, 0, 0, 0, 1,
  1.801, 1.211, 0.015, 0.000, 0, 0, 0, 0, 1,
  -0.207, 1.191, 0.066, 0.033, 0, 0, 0, 0, 1,
  -0.215, 1.291, -0.087, 0.029, 0, 0, 0, 0, 1,
  0.259, 0.875, 0.177, 0.072, 0, 0, 0, 0, 1,
  -0.423, 0.942, 0.064, 0.094, 0, 0, 0, 0, 1,
  0.113, 0.795, 0.124, 0.110, 0, 0, 0, 0, 1,
  1.030, 1.525, 0.200, 0.076, 0, 0, 0, 0, 1,
  0.140, 1.209, 0.082, 0.148, 0, 0, 0, 0, 1,
  0.429, 1.480, -0.008, 0.061, 0, 0, 0, 0, 1,
  0.089, 0.785, -0.065, 0.018, 0, 0, 0, 0, 1,
  -0.516, 1.013, 0.016, 0.023, 0, 0, 0, 0, 1,
  0.143, 1.315, -0.011, 0.136, 0, 0, 0, 0, 1,
  0.347, 0.733, -0.121, 0.041, 0, 0, 0, 0, 1,
  -0.074, 0.869, 0.013, 0.026, 0, 0, 0, 0, 1,
  0.630, 1.484, -0.001, 0.000, 0, 0, 0, 0, 1),
  nrow=23, ncol=9, byrow=TRUE)

theta <- rnorm(NSubj)
data<-genFMPData(NSubj = NSubj, bParam = b, theta = theta, seed = 345)$data

erfItem1 <- erf(theta, data, whichItem = 1, min = -3, max = 3, Ncuts = 12)

plot( erfItem1$centers, erfItem1$probs, type="b",
      main="Empirical Response Function",
      xlab = expression(theta),
      ylab="Probability",
      cex.lab=1.5)

```

---

faAlign

---

Align the columns of two factor loading matrices

---

## Description

Align factor loading matrices across solutions using the Hungarian algorithm to locate optimal matches. faAlign will match the factors of F2 (the input matrix) to those in F1 (the target matrix) to minimize a least squares discrepancy function or to maximize factor congruence coefficients (i.e., vector cosines).

## Usage

```
faAlign(F1, F2, Phi2 = NULL, MatchMethod = "LS")
```

**Arguments**

F1	target Factor Loadings Matrix.
F2	input Factor Loadings Matrix. F2 will be aligned with the target matrix, F1.
Phi2	optional factor correlation matrix for F2 (default = NULL).
MatchMethod	"LS" (Least Squares) or "CC" (congruence coefficients).

**Value**

F2	re-ordered and reflected loadings of F2.
Phi2	reordered and reflected factor correlations.
FactorMap	a 2 x k matrix (where k is the number of columns of F1) structured such that row 1: the original column order of F2; row 2: the sorted column order of F2.
UniqueMatch	(logical) indicates whether a unique match was found.
MatchMethod	"LS" (least squares) or "CC" (congruence coefficients, i.e., cosines).
CC	Congruence coefficients for the matched factors.
LS	Root-mean-squared-deviations (least squares criterion) for the matched factors.

**Note**

The Hungarian algorithm is implemented with the clue (Cluster Ensembles, Hornik, 2005) package. See Hornik K (2005). A CLUE for CLUster Ensembles. *Journal of Statistical Software*, 14(12). doi: 10.18637/jss.v014.i12 (URL: <http://doi.org/10.18637/jss.v014.i12>).

**Author(s)**

Niels Waller

**References**

- Kuhn, H. W. (1955). The Hungarian Method for the assignment problem. *Naval Research Logistics Quarterly*, 2, 83-97.
- Kuhn, H. W. (1956). Variants of the Hungarian method for assignment problems. *Naval Research Logistics Quarterly*, 3, 253-258.
- Papadimitriou, C. & Steiglitz, K. (1982). Combinatorial Optimization: Algorithms and Complexity. Englewood Cliffs: Prentice Hall.

**Examples**

```
# This example demonstrates the computation of
# non-parametric bootstrap confidence intervals
# for rotated factor loadings.

library(GPARotation)

data(HS9Var)

HS9 <- HS9Var[HS9Var$school == "Grant-White",7:15]

# Compute an R matrix for the HSVar9 Mental Abilities Data
R.HS9 <- cor(HS9)
```

```

varnames <- c( "vis.per", "cubes",
               "lozenges", "paragraph.comp",
               "sentence.comp", "word.mean",
               "speed.add", "speed.count.dots",
               "speed.discr")

# Extract and rotate a 3-factor solution
# via unweighted least squares factor extraction
# and oblimin rotation.

NFac <- 3
NVar <- 9
B <- 200      # Number of bootstrap samples
NSubj <- nrow(HS9)

# Unrotated 3 factor uls solution
F3.uls <- fals(R = R.HS9, nfactors = NFac)

# Rotate via oblimin
F3.rot <- oblimin(F3.uls$loadings,
                  gam = 0,
                  normalize = FALSE)

F3.loadings <- F3.rot$loadings
F3.phi <- F3.rot$Phi

# Reflect factors so that salient loadings are positive
Dsgn <- diag(sign(colSums(F3.loadings^3)))
F3.loadings <- F3.loadings %**% Dsgn
F3.phi <- Dsgn %**% F3.phi %**% Dsgn

rownames(F3.loadings) <- varnames
colnames(F3.loadings) <- paste0("f", 1:3)
colnames(F3.phi) <- rownames(F3.phi) <- paste0("f", 1:3)

cat("\nOblimin rotated factor loadings for 9 Mental Abilities Variables")
print( round(F3.loadings, 2))

cat("\nFactor correlation matrix")
print( round( F3.phi, 2))

# Declare variables to hold bootstrap output
Flist <- Philist <- as.list(rep(0, B))
UniqueMatchVec <- rep(0, B)
rows <- 1:NSubj

# Analyze bootstrap samples and record results
for(i in 1:B){
  cat("\nWorking on sample ", i)
  set.seed(i)

  # Create bootstrap samples
  bsRows <- sample(rows, NSubj, replace= TRUE)
  Fuls <- fals(R = cor(HS9[bsRows, ]), nfactors = NFac)

```

```

# rotated loadings
Fboot <- oblimin(Fuls$loadings,
                gam = 0,
                normalize = FALSE)

out <- faAlign(F1 = F3.loadings,
              F2 = Fboot$loadings,
              MatchMethod = "LS")

Flist[[i]] <- out$F2 # aligned version of Fboot$loadings
UniqueMatchVec[i] <- out$UniqueMatch
}

cat("\nNumber of Unique Matches: ",
    100*round(mean(UniqueMatchVec),2), "%\n")

# Make a 3D array from list of matrices
arr <- array( unlist(Flist) , c(NVar, NFac, B) )

# Get quantiles of factor elements over third dimension (samples)
F95 <- apply( arr , 1:2 , quantile, .975 )
F05 <- apply( arr , 1:2 , quantile, .025 )
Fse <- apply( arr , 1:2, sd )

cat("\nUpper Bound 95% CI\n")
print( round(F95,3))
cat("\n\nLower Bound 95% CI\n")
print( round(F05,3))

# plot distribution of bootstrap estimates
# for example element
hist(arr[5,1,], xlim=c(.4,1),
     main = "Bootstrap Distribution for F[5,1]",
     xlab = "F[5,1]")

print(round( F3.loadings, 2))
cat("\nStandard Errors")
print( round( Fse, 2))

```

---

fals

---

*Unweighted least squares factor analysis*


---

## Description

Unweighted least squares factor analysis

## Usage

```

fals(R,
     nfactors,
     TreatHeywood = TRUE)

```

**Arguments**

R	Input correlation matrix.
nfactors	Number of factors to extract.
TreatHeywood	If TreatHeywood = TRUE then a penalized least squares function is used to bound the commonality estimates below 1.0. Default(TreatHeywood = TRUE).

**Value**

loadings	Unrotated factor loadings. If a Heywood case is present in the initial solution then the model is re-estimated via non-iterated principal axes with $\max(\text{rij}^2)$ as fixed communaility (h2) estimates.
h2	Vector of final commonality estimates.
uniqueness	Vector of factor uniquenesses, i.e. $(1 - h2)$ .
Heywood	(logical) TRUE if a Heywood case was produced in the LS solution.
TreatHeywood	(logical) Value of the TreatHeywood argument.
converged	(logical) TRUE if all values of the gradient are sufficiently close to zero.
MaxAbsGrad	The maximum absolute value of the gradient at the solution.

**Author(s)**

Niels Waller

**Examples**

```
Rbig <- fungible::rcor(120)
out1 <- fals(R = Rbig,
             nfactors = 2,
             TreatHeywood = TRUE)
```

---

faMAP	<i>Velicer's minimum partial correlation method for determining the number of major components for a principal components analysis or a factor analysis</i>
-------	---

---

**Description**

Uses Velicer's MAP (i.e., matrix of partial correlations) procedure to determine the number of components from a matrix of partial correlations.

**Usage**

```
faMAP(R, max.fac = 8, Print = TRUE, Plot = TRUE)
```

**Arguments**

R	input data in the form of a correlation matrix.
max.fac	maximum number of dimensions to extract.
Print	(logical) Print = TRUE will print complete results.
Plot	(logical) Plot = TRUE will plot the MAP values.

**Value**

MAP	Minimum partial correlations
MAP4	Minimum partial correlations
fm	average of the squared partial correlations after the first m components are partialled out.
fm4	see Velicer, Eaton, & Fava, 2000.
PlotAvgSq	A saved object of the original MAP plot (based on the average squared partial r's.)
PlotAvg4th	A saved object of the revised MAP plot (based on the average 4th power of the partial r's.)

**Author(s)**

Niels Waller

**References**

Velicer, W. (1976). Determining the number of components from the matrix of partial correlations. *Psychometrika*, 41(3):321–327.

Velicer, W. F., Eaton, C. A., & Fava, J. L. (2000). Construct explication through factor or component analysis: A review and evaluation of alternative procedures for determining the number of factors or components. In R. D. Goffin & E. Helmes (Eds.). *Problems and Solutions in Human Assessment: Honoring Douglas N. Jackson at Seventy* (pp. 41-71. Boston, MA: Kluwer Academic.

**Examples**

```
# Harman's data (1967, p 80)
# R = matrix(c(
# 1.000, .846, .805, .859, .473, .398, .301, .382,
# .846, 1.000, .881, .826, .376, .326, .277, .415,
# .805, .881, 1.000, .801, .380, .319, .237, .345,
# .859, .826, .801, 1.000, .436, .329, .327, .365,
# .473, .376, .380, .436, 1.000, .762, .730, .629,
# .398, .326, .319, .329, .762, 1.000, .583, .577,
# .301, .277, .237, .327, .730, .583, 1.000, .539,
# .382, .415, .345, .365, .629, .577, .539, 1.000), 8,8)

F <- matrix(c( .4, .1, .0,
               .5, .0, .1,
               .6, .03, .1,
               .4, -.2, .0,
               0, .6, .1,
               .1, .7, .2,
               .3, .7, .1,
               0, .4, .1,
               0, 0, .5,
               .1, -.2, .6,
               .1, .2, .7,
               -.2, .1, .7), 12, 3)

R <- F %*% t(F)
diag(R) <- 1

faMAP(R, max.fac = 8, Print = TRUE, Plot = TRUE)
```



faSort

*Sort a factor loadings matrix***Description**

faSort takes an unsorted factor pattern or structure matrix and returns a sorted matrix with (possibly) reflected columns. Sorting is done such that variables that load on a common factor are grouped together for ease of interpretation.

**Usage**

```
faSort(fmat, phi = NULL, salient = .25, reflect = TRUE)
```

**Arguments**

fmat	factor loadings (pattern or structure) matrix.
phi	factor correlation matrix. Default = NULL. If reflect = TRUE then phi will be corrected to match the new factor orientations.
salient	factor markers with loadings $\geq \text{abs}(\text{salient})$ will be saved in the markers list. Note that a variable can be a marker of more than one factor.
reflect	(logical) if reflect = TRUE then the factors will be reflected such that salient loadings are mostly positive.

**Value**

loadings	sorted factor loadings matrix.
phi	reflected factor correlation matrix when phi is given as an argument.
markers	A list of factor specific markers with loadings $\geq \text{abs}(\text{salient})$ . Markers are sorted by the absolute value of the salient factor loadings.
sortOrder	sorted row numbers.
SEmat	The SEmat is a so-called Start-End matrix that lists the first (start) and last (end) row for each factor in the sorted pattern matrix.

**Author(s)**

Niels Waller

**See Also**

[fals](#)

**Examples**

```
set.seed(123)
F <- matrix( c( .5,  0,
                .6,  0,
                0,  .6,
                .6,  0,
                0,  .5,
                .7,  0,
```

```

      0, .7,
      0, .6), nrow = 8, ncol = 2, byrow=TRUE)

Rex1 <- F %*% t(F); diag(Rex1) <- 1

Items <- c("1. I am often tense.\n",
          "2. I feel anxious much of the time.\n",
          "3. I am a naturally curious individual.\n",
          "4. I have many fears.\n",
          "5. I read many books each year.\n",
          "6. My hands perspire easily.\n",
          "7. I have many interests.\n",
          "8. I enjoy learning new words.\n")

exampleOut <- fals(R = Rex1, nfactors = 2)

# Varimax rotation
Fload <- varimax(exampleOut$loadings)$loadings[]

# Add some row labels
rownames(Fload) <- paste0("V", 1:nrow(Fload))

cat("\nUnsorted fator loadings\n")
print(round( Fload, 2) )

# Sort items and reflect factors
out1 <- faSort(fmat = Fload,
              salient = .25,
              reflect = TRUE)

FloadSorted <- out1$loadings

cat("\nSorted fator loadings\n")
print(round( FloadSorted, 2) )

# Print sorted items
cat("\n Items sorted by Factor\n")
cat("\n",Items[out1$sortOrder])

```

FMP

*Estimate the coefficients of a filtered monotonic polynomial IRT model***Description**

Estimate the coefficients of a filtered monotonic polynomial IRT model.

**Usage**

```
FMP(data, thetaInit, item, startvals, k, eps = 1e-06)
```

**Arguments**

data	N(subjects)-by-p(items) matrix of 0/1 item response data.
thetaInit	Initial theta ( $\theta$ ) surrogates (e.g., calculated by <a href="#">svdNorm</a> ).

item	Item number for coefficient estimation.
startvals	Start values for function minimization. Start values are in the gamma metric (see Liang & Browne, 2015)
k	Order of monotonic polynomial = $2k+1$ (see Liang & Browne, 2015). $k$ can equal 0, 1, 2, or 3.
eps	Step size for gradient approximation, default = $1e-6$ . If a convergence failure occurs during function optimization reducing the value of eps will often produce a converged solution.

## Details

As described by Liang and Browne (2015), the filtered polynomial model (FMP) is a quasi-parametric IRT model in which the IRF is a composition of a logistic function and a polynomial function,  $m(\theta)$ , of degree  $2k + 1$ . When  $k = 0$ ,  $m(\theta) = b_0 + b_1\theta$  (the slope intercept form of the 2PL). When  $k = 1$ ,  $2k + 1$  equals 3 resulting in  $m(\theta) = b_0 + b_1\theta + b_2\theta^2 + b_3\theta^3$ . Acceptable values of  $k = 0, 1, 2, 3$ . According to Liang and Browne, the "FMP IRF may be used to approximate any IRF with a continuous derivative arbitrarily closely by increasing the number of parameters in the monotonic polynomial" (2015, p. 2) The FMP model assumes that the IRF is monotonically increasing, bounded by 0 and 1, and everywhere differentiable with respect to theta (the latent trait).

## Value

b	Vector of polynomial coefficients.
gamma	Polynomial coefficients in gamma metric (see Liang & Browne, 2015).
FHAT	Function value at convergence.
counts	Number of function evaluations during minimization (see optim documentation for further details).
AIC	Pseudo scaled Akaike Information Criterion (AIC). Candidate models that produce the smallest AIC suggest the optimal number of parameters given the sample size. Scaling is accomplished by dividing the non-scaled AIC by sample size.
BIC	Pseudo scaled Bayesian Information Criterion (BIC). Candidate models that produce the smallest BIC suggest the optimal number of parameters given the sample size. Scaling is accomplished by dividing the non-scaled BIC by sample size.
convergence	Convergence = 0 indicates that the optimization algorithm converged; convergence=1 indicates that the optimization failed to converge.

## Author(s)

Niels Waller

## References

Liang, L. & Browne, M. W. (2015). A quasi-parametric method for fitting flexible item response functions. *Journal of Educational and Behavioral Statistics*, 40, 5–34.

## Examples

```
## Not run:
## In this example we will generate 2000 item response vectors
## for a k = 1 order filtered polynomial model and then recover
## the estimated item parameters with the FMP function.

k <- 1 # order of polynomial

NSubjects <- 2000

## generate a sample of 2000 item response vectors
## for a k = 1 FMP model using the following
## coefficients
b <- matrix(c(
  #b0    b1      b2      b3    b4    b5    b6    b7    k
  1.675, 1.974, -0.068, 0.053, 0, 0, 0, 0, 1,
  1.550, 1.805, -0.230, 0.032, 0, 0, 0, 0, 1,
  1.282, 1.063, -0.103, 0.003, 0, 0, 0, 0, 1,
  0.704, 1.376, -0.107, 0.040, 0, 0, 0, 0, 1,
  1.417, 1.413, 0.021, 0.000, 0, 0, 0, 0, 1,
  -0.008, 1.349, -0.195, 0.144, 0, 0, 0, 0, 1,
  0.512, 1.538, -0.089, 0.082, 0, 0, 0, 0, 1,
  0.122, 0.601, -0.082, 0.119, 0, 0, 0, 0, 1,
  1.801, 1.211, 0.015, 0.000, 0, 0, 0, 0, 1,
  -0.207, 1.191, 0.066, 0.033, 0, 0, 0, 0, 1,
  -0.215, 1.291, -0.087, 0.029, 0, 0, 0, 0, 1,
  0.259, 0.875, 0.177, 0.072, 0, 0, 0, 0, 1,
  -0.423, 0.942, 0.064, 0.094, 0, 0, 0, 0, 1,
  0.113, 0.795, 0.124, 0.110, 0, 0, 0, 0, 1,
  1.030, 1.525, 0.200, 0.076, 0, 0, 0, 0, 1,
  0.140, 1.209, 0.082, 0.148, 0, 0, 0, 0, 1,
  0.429, 1.480, -0.008, 0.061, 0, 0, 0, 0, 1,
  0.089, 0.785, -0.065, 0.018, 0, 0, 0, 0, 1,
  -0.516, 1.013, 0.016, 0.023, 0, 0, 0, 0, 1,
  0.143, 1.315, -0.011, 0.136, 0, 0, 0, 0, 1,
  0.347, 0.733, -0.121, 0.041, 0, 0, 0, 0, 1,
  -0.074, 0.869, 0.013, 0.026, 0, 0, 0, 0, 1,
  0.630, 1.484, -0.001, 0.000, 0, 0, 0, 0, 1),
  nrow=23, ncol=9, byrow=TRUE)

ex1.data <- genFMPData(NSubj = NSubjects, bParams = b, seed = 345)$data

## number of items in the data matrix
NItems <- ncol(ex1.data)

# compute (initial) surrogate theta values from
# the normed left singular vector of the centered
# data matrix
thetaInit <- svdNorm(ex1.data)

## earlier we defined k = 1
if(k == 0) {
  startVals <- c(1.5, 1.5)
```

```

        bmat <- matrix(0, NItems, 6)
        colnames(bmat) <- c(paste("b", 0:1, sep = ""), "FHAT", "AIC", "BIC", "convergence")
    }
    if(k == 1) {
        startVals <- c(1.5, 1.5, .10, .10)
        bmat <- matrix(0, NItems, 8)
        colnames(bmat) <- c(paste("b", 0:3, sep = ""), "FHAT", "AIC", "BIC", "convergence")
    }
    if(k == 2) {
        startVals <- c(1.5, 1.5, .10, .10, .10, .10)
        bmat <- matrix(0, NItems, 10)
        colnames(bmat) <- c(paste("b", 0:5, sep = ""), "FHAT", "AIC", "BIC", "convergence")
    }
    if(k == 3) {
        startVals <- c(1.5, 1.5, .10, .10, .10, .10, .10, .10)
        bmat <- matrix(0, NItems, 12)
        colnames(bmat) <- c(paste("b", 0:7, sep = ""), "FHAT", "AIC", "BIC", "convergence")
    }
}

# estimate item parameters and fit statistics
for(i in 1:NItems){
    out <- FMP(data = ex1.data, thetaInit, item = i, startvals = startVals, k = k)
    Nb <- length(out$b)
    bmat[i,1:Nb] <- out$b
    bmat[i,Nb+1] <- out$FHAT
    bmat[i,Nb+2] <- out$AIC
    bmat[i,Nb+3] <- out$BIC
    bmat[i,Nb+4] <- out$convergence
}

# print output
print(bmat)

## End(Not run)

```

---

FMPMonotonicityCheck    *Utility function for checking FMP monotonicity*

---

## Description

Utility function for checking whether candidate FMP coefficients yield a monotonically increasing polynomial.

## Usage

```
FMPMonotonicityCheck(b, lower = -20, upper = 20, PLOT = FALSE)
```

## Arguments

b	A vector of 8 polynomial coefficients ( $b$ ) for $m(\theta) = b_0 + b_1\theta + b_2\theta^2 + b_3\theta^3 + b_4\theta^4 + b_5\theta^5 + b_6\theta^6 + b_7\theta^7$ .
lower, upper	$\theta$ bounds for monotonicity check.
PLOT	Logical (default = FALSE). If PLOT = TRUE the function will plot the original polynomial function for $\theta$ between lower and upper.

**Value**

increasing	Logical indicating whether function is monotonically increasing.
minDeriv	Minimum value of the derivative for the polynomial.
minTheta	Value of $\theta$ at derivative minimum.

**Author(s)**

Niels Waller

**Examples**

```
## A set of candidate coefficients for an FMP model.
## These coefficients fail the test and thus
## should not be used with genFMPdata to generate
## item response data that are consistent with an
## FMP model.
b <- c(1.21, 1.87, -1.02, 0.18, 0.18, 0, 0, 0)
FMPMonotonicityCheck(b)
```

---

fungible

*Generate Fungible Regression Weights*

---

**Description**

Generate fungible weights for OLS Regression Models.

**Usage**

```
fungible(R.X, rxy, r.yhata.yhatb, sets, print = TRUE)
```

**Arguments**

R.X	p x p Predictor correlation matrix.
rxy	p x 1 Vector of predictor-criterion correlations.
r.yhata.yhatb	Correlation between least squares (yhatb) and alternate-weight (yhata) composites.
sets	Number of returned sets of fungible weights.
print	Logical, if TRUE then print 5-point summaries of alternative weights.

**Value**

a	Number of sets x p matrix of fungible weights.
k	Number of sets x p matrix of k weights.
b	p x 1 vector of LS weights.
u	p x 1 vector of u weights.
r.yhata.yhatb	Correlation between yhata and yhatb.
r.y.yhatb	Correlation between y and yhatb.
cov.a	Expected covariance matrix for a.
cor.a	Expected correlation matrix for a.

**Author(s)**

Niels Waller

**References**

Waller, N. (2008). Fungible weights in multiple regression. *Psychometrika*, 73, 69–703.

**Examples**

```
## Predictor correlation matrix
R.X <- matrix(c(1.00, .56, .77,
               .56, 1.00, .73,
               .77, .73, 1.00), 3, 3)

## vector of predictor-criterion correlations
rxy <- c(.39, .34, .38)

## OLS standardized regression coefficients
b <- solve(R.X) %*% rxy

## Coefficient of determination (Rsqr)
OLSRSQ <- t(b) %*% R.X %*% b

## theta controls the correlation between
## yhatb: predicted criterion scores using OLS coefficients
## yhata: predicted criterion scores using alternate weights
theta <- .01

## desired correlation between yhata and yhatb
r.yhata.yhatb <- sqrt( 1 - (theta)/OLSRSQ)

## number of returned sets of fungible weight vectors
Nsets <- 50

output <- fungible(R.X, rxy, r.yhata.yhatb, sets = Nsets, print = TRUE)
```

---

fungibleExtrema

*Locate Extrema of Fungible Regression Weights*


---

**Description**

Locate extrema of fungible regression weights.

**Usage**

```
fungibleExtrema(R.X, rxy, r.yhata.yhatb, Nstarts, MaxMin)
```

**Arguments**

R.X	p x p Predictor variable correlation matrix.
rx	p x 1 Vector of predictor-criterion correlations.
r.yhata.yhatb	Correlation between least squares (yhatb) and alternate-weight (yhata) composites.
Nstarts	Maximum number of (max) minimizations from random starting configurations.
MaxMin	Character: "Max" = maximize cos(a,b); "Min" = minimize cos(a,b).

**Value**

cos.ab	cosine between OLS and alternate weights.
a	extrema of fungible weights.
k	k weights.
z	z weights: a normalized random vector.
b	OLS weights.
u	p x 1 vector of u weights.
r.yhata.yhatb	Correlation between yhata and yhatb.
r.y.yhatb	Correlation between y and yhatb.
gradient	Gradient of converged solution.

**Author(s)**

Niels Waller and Jeff Jones

**References**

- Koopman, R. F. (1988). On the sensitivity of a composite to its weights. *Psychometrika*, 53(4), 547–552.
- Waller, N. & Jones, J. (2009). Locating the extrema of fungible regression weights in multiple regression. *Psychometrika*, 74, 589–602.

**Examples**

```
## Not run:
## Example
## This is Koopman's Table 2 Example

R.X <- matrix(c(1.00, .69, .49, .39,
               .69, 1.00, .38, .19,
               .49, .38, 1.00, .27,
               .39, .19, .27, 1.00),4,4)

b <- c(.39, .22, .02, .43)
rx <- R.X %*% b

OLSRSQ <- t(b) %*% R.X %*% b

## theta <- .02
```



```
## r.yhata.yhatb <- sqrt( 1 - (theta)/OLSRSQ)

r.yhata.yhatb <- .90
set.seed(5)
output <- fungibleExtrema(R.X, rxy, r.yhata.yhatb, Nstarts = 500,
                          MaxMin = "Min")

## Scale to replicate Koopman
a <- output$a
a.old <- a
aRa <- t(a) %*% R.X %*% a

## Scale a such that a' R a = .68659
## vc = variance of composite
vc <- aRa
## sf = scale factor
sf <- .68659/vc
a <- as.numeric(sqrt(sf)) * a
cat("\nKoopman Scaling\n")
print(round(a,2))

## End(Not run)
```

---

fungibleL

---

*Generate Fungible Logistic Regression Weights*


---

## Description

Generate fungible weights for Logistic Regression Models.

## Usage

```
fungibleL(X, y, Nsets = 1000, method = "LLM", RsqDelta = NULL,
          rLaLb = NULL, s = .3, Print = TRUE)
```

## Arguments

X	An n by nvar matrix of predictor scores without the leading column of ones.
y	An n by 1 vector of dichotomous criterion scores.
Nsets	The desired number of fungible coefficient vectors.
method	Character: "LLM" = Log-Likelihood method. "EM" = Ellipsoid Method. Default: method = "LLM".
RsqDelta	The desired decrement in the pseudo-R-squared - used when method = "LLM".
rLaLb	The desired correlation between the logits - used when method = "EM".
s	Scale factor for random deviates. s controls the range of random start values for the optimization routine. Recommended $0 \leq s < 1$ . Default: s = 0.3.
Print	Boolean (TRUE/FALSE) for printing output summary.

## Details

fungibleL provides two methods for evaluating parameter sensitivity in logistic regression models by computing fungible logistic regression weights. For additional information on the underlying theory of these methods see Jones and Waller (in press).

**Value**

model	A glm model object.
call	The function call to glm().
ftable	A data frame with the mle estimates and the minimum and maximum fungible coefficients.
lnLML	The maximum likelihood log likelihood value.
lnLf	The decremented, fungible log likelihood value.
pseudoRsqr	The pseudo R-squared.
fungibleRsqr	The fungible pseudo R-squared.
fungiblea	The Nsets by Nvar + 1 matrix of fungible (alternate) coefficients.
rLaLb	The correlation between the logits.
maxPosCoefChange	The maximum positive change in a single coefficient holding all other coefficients constant.
maxNegCoefChange	The maximum negative change in a single coefficient holding all other coefficients constant.

**Author(s)**

Jeff Jones and Niels Waller

**References**

Jones, J. A. & Waller, N. G. (in press). Fungible weights in logistic regression. *Psychological Methods*.

**Examples**

```
# Example: Low Birth Weight Data from Hosmer Jr, D. W. & Lemeshow, S.(2000).
# low : low birth rate (0 >= 2500 grams, 1 < 2500 grams)
# race: 1 = white, 2 = black, 3 = other
# ftv : number of physician visits during the first trimester

library(MASS)
attach(birthwt)

race <- factor(race, labels = c("white", "black", "other"))
predictors <- cbind(lwt, model.matrix(~ race)[, -1])

# compute mle estimates
BWght.out <- glm(low ~ lwt + race, family = "binomial")

# compute fungible coefficients
fungible.LLM <- fungibleL(X = predictors, y = low, method = "LLM",
                        Nsets = 10, RsqDelta = .005, s = .3)

# Compare with Table 2.3 (page 38) Hosmer Jr, D. W. & Lemeshow, S.(2000).
# Applied logistic regression. New York, Wiley.

print(summary(BWght.out))
print(fungible.LLM$call)
```

```

print(fungible.LLM$fhtable)
cat("\nMLE log likelihood      = ", fungible.LLM$nLML,
    "\nfungible log likelihood = ", fungible.LLM$nLf)
cat("\nPseudo Rsq              = ", round(fungible.LLM$pseudoRsq, 3))
cat("\nfungible Pseudo Rsq      = ", round(fungible.LLM$fungibleRsq, 3))

fungible.EM <- fungibleL(X = predictors, y = low, method = "EM" ,
                        Nsets = 10, rLaLb = 0.99)

print(fungible.EM$call)
print(fungible.EM$fhtable)

cat("\nrLaLb = ", round(fungible.EM$rLaLb, 3))

```

fungibleR

*Generate Fungible Correlation Matrices***Description**

Generate fungible correlation matrices. For a given vector of standardized regression coefficients, Beta, and a user-define R-squared value, Rsq, find predictor correlation matrices, R, such that Beta' R Beta = Rsq. The size of the smallest eigenvalue (Lp) of R can be defined.

**Usage**

```
fungibleR(R, Beta, Lp = .00, eps = 1e-08, Print.Warnings = TRUE)
```

**Arguments**

R	A p x p predictor correlation matrix.
Beta	A p x 1 vector of standardized regression coefficients.
Lp	Controls the size of the smallest eigenvalue of RstarLp.
eps	Convergence criterion.
Print.Warnings	Logical, default = TRUE. When TRUE, convergence failures are printed.

**Value**

R	Any input correlation matrix that satisfies Beta' R Beta = Rsq.
Beta	Input vector of std reg coefficients.
Rstar	A random fungible correlation matrix.
RstarLp	A fungible correlation matrix with a fixed minimum eigenvalue (RstarLp can be PD, PSD, or ID).
s	Scaling constant for Rstar.
sLp	Scaling constant for RstarLp.
Delta	Vector in the null space of vecp(Beta Beta').
Q	Left null space of Beta.
FrobNorm	Frobenius norm   R - Rstar  _F.
FrobNormLp	Frobenius norm   R - RstarLp  _F given random Delta.
converged	An integer code. 0 indicates successful completion.

**Author(s)**

Niels Waller

**References**

Waller, N. (2016). Fungible Correlation Matrices: A method for generating nonsingular, singular, and improper correlation matrices for Monte Carlo research. *Multivariate Behavioral Research*.

**Examples**

```
library(fungible)

## ===== Example 1 =====
## Generate 5 random PD fungible R matrices
## that are consistent with a user-defined predictive
## structure: B' Rxx B = .30

set.seed(246)
## Create a 5 x 5 correlation matrix, R, with all r_ij = .25
R.ex1 <- matrix(.25, 5, 5)
diag(R.ex1) <- 1

## create a 5 x 1 vector of standardized regression coefficients,
## Beta.ex1
Beta.ex1 <- c(-.4, -.2, 0, .2, .4)
cat("\nModel Rsq = ", t(Beta.ex1) %*% R.ex1 %*% Beta.ex1)

## Generate fungible correlation matrices, Rstar, with smallest
## eigenvalues > 0.

Rstar.list <- list(rep(99,5))
i <- 0
while(i <= 5){
  out <- fungibleR(R = R.ex1, Beta = Beta.ex1, Lp = 1e-8, eps = 1e-8,
                  Print.Warnings = TRUE)
  if(out$converged==0){
    i <- i + 1
    Rstar.list[[i]] <- out$Rstar
  }
}

## Check Results
cat("\n *** Check Results ***")
for(i in 1:5){
  cat("\n\n\n+++++")
  cat("\nRstar", i, "\n")
  print(round(Rstar.list[[i]], 2),)
  cat("\neigenvalues of Rstar", i, "\n")
  print(eigen(Rstar.list[[i]])$values)
  cat("\nBeta' Rstar", i, "Beta = ",
      t(Beta.ex1) %*% Rstar.list[[i]] %*% Beta.ex1)
}
```

```

## ===== Example 2 =====
## Generate a PD fungible R matrix with a fixed smallest
## eigenvalue (Lp).

## Create a 5 x 5 correlation matrix, R, with all r_ij = .5
R <- matrix(.5, 5, 5)
diag(R) <- 1

## create a 5 x 1 vector of standardized regression coefficients, Beta,
## such that Beta_i = .1 for all i
Beta <- rep(.1, 5)

## Generate fungible correlation matrices (a) Rstar and (b) RstarLp.
## Set Lp = 0.12345678 so that the smallest eigenvalue (Lp) of RstarLp
## = 0.12345678
out <- fungibleR(R, Beta, Lp = 0.12345678, eps = 1e-10, Print.Warnings = TRUE)

## print R
cat("\nR: a user-specified seed matrix")
print(round(out$R,3))

## Rstar
cat("\nRstar: A random fungible correlation matrix for R")
print(round(out$Rstar,3))

cat("\nCoefficient of determination when using R\n")
print( t(Beta) %*% R %*% Beta )

cat("\nCoefficient of determination when using Rstar\n")
print( t(Beta) %*% out$Rstar %*% Beta)

## Eigenvalues of R
cat("\nEigenvalues of R\n")
print(round(eigen(out$R)$values, 9))

## Eigenvalues of Rstar
cat("\nEigenvalues of Rstar\n")
print(round(eigen(out$Rstar)$values, 9))

## What is the Frobenius norm (Euclidean distance) between
## R and Rstar
cat("\nFrobenious norm ||R - Rstar||\n")
print( out$FrobNorm)

## RstarLp is a random fungible correlation matrix with
## a fixed smallest eigenvalue of 0.12345678
cat("\nRstarLp: a random fungible correlation matrix with a user-defined
smallest eigenvalue\n")
print(round(out$RstarLp, 3))

## Eigenvalues of RstarLp
cat("\nEigenvalues of RstarLp")
print(eigen(out$RstarLp)$values, digits = 9)

cat("\nCoefficient of determination when using RstarLp\n")
print( t(Beta) %*% out$RstarLp %*% Beta)

```

```

## Check function convergence
if(out$converged) print("Failed to converge")

## ===== Example 3 =====
## This examples demonstrates how fungibleR can be used
## to generate improper correlation matrices (i.e., pseudo
## correlation matrices with negative eigenvalues).
library(fungible)

## We desire an improper correlation matrix that
## is close to a user-supplied seed matrix. Create an
## interesting seed matrix that reflects a Big Five
## factor structure.

set.seed(123)
minCrossLoading <- -.2
maxCrossLoading <- .2
F1 <- c(rep(.6,5),runif(20,minCrossLoading, maxCrossLoading))
F2 <- c(runif(5,minCrossLoading, maxCrossLoading), rep(.6,5),
        runif(15,minCrossLoading, maxCrossLoading))
F3 <- c(runif(10,minCrossLoading,maxCrossLoading), rep(.6,5),
        runif(10,minCrossLoading,maxCrossLoading) )
F4 <- c(runif(15,minCrossLoading,maxCrossLoading), rep(.6,5),
        runif(5,minCrossLoading,maxCrossLoading))
F5 <- c(runif(20,minCrossLoading,maxCrossLoading), rep(.6,5))
FacMat <- cbind(F1,F2,F3,F4,F5)
R.bfi <- FacMat %*% t(FacMat)
diag(R.bfi) <- 1

## Set Beta to a null vector to inform fungibleR that we are
## not interested in placing constraints on the predictive structure
## of the fungible R matrices.
Beta <- rep(0, 25)

## We seek a NPD fungible R matrix that is close to the bfi seed matrix.
## To find a suitable matrix we generate a large number (e.g., 50000)
## fungible R matrices. For illustration purposes I will set Nmatrices
## to a smaller number: 10.
Nmatrices<-10

## Initialize a list to contain the Nmatrices fungible R objects
RstarLp.list <- as.list( rep(0, Nmatrices) )
## Initialize a vector for the Nmatrices Frobenius norms ||R - RstarLp||
FrobLp.vec <- rep(0, Nmatrices)

## Constraint the smallest eigenvalue of RStarLp by setting
## Lp = -.1 (or any suitably chosen user-defined value).

## Generate Nmatrices fungibleR matrices and identify the NPD correlation
## matrix that is "closest" (has the smallest Frobenious norm) to the bfi
## seed matrix.
BestR.i <- 0
BestFrob <- 99
i <- 0

```

```

set.seed(1)
while(i < Nmatrices){
  out<-fungibleR(R = R.bfi, Beta, Lp = -.1, eps=1e-10)
  ## retain solution if algorithm converged
  if(out$converged == 0)
  {
    i<- i + 1
    ## print progress
    cat("\nGenerating matrix ", i, " Current minimum ||R - RstarLp|| = ",BestFrob)
    tmp <- FrobLp.vec[i] <- out$FrobNormLp #Frobenious Norm ||R - RstarLp||
    RstarLp.list[[i]]<-out$RstarLp
    if( tmp < BestFrob )
    {
      BestR.i <- i      # matrix with lowest ||R - RstarLp||
      BestFrob <- tmp   # value of lowest ||R - RstarLp||
    }
  }
}

# CloseR is an improper correlation matrix that is close to the seed matrix.
CloseR<-RstarLp.list[[BestR.i]]

plot(1:25, eigen(R.bfi)$values,
     type = "b",
     lwd = 2,
     main = "Scree Plots for R and RstarLp",
     cex.main = 1.5,
     ylim = c(-.2,6),
     ylab = "Eigenvalues",
     xlab = "Dimensions")
points(1:25,eigen(CloseR)$values,
      type = "b",
      lty = 2,
      lwd = 2,
      col = "red")
abline(h = 0, col = "grey")
legend(legend=c(expression(paste(lambda[i]~" of R",sep = "")),
                      expression(paste(lambda[i]~" of RstarLp",sep = ""))),
      lty=c(1,2),
      x = 17,y = 5.75,
      cex = 1.5,
      col=c("black", "red"),
      text.width = 5.5,
      lwd = 2)

```

FUP

---

*Estimate the coefficients of a filtered unconstrained polynomial IRT model*

---

## Description

Estimate the coefficients of a filtered unconstrained polynomial IRT model.

**Usage**

```
FUP(data, thetaInit, item, startvals, k)
```

**Arguments**

data	N(subjects)-by-p(items) matrix of 0/1 item response data.
thetaInit	Initial theta surrogates (e.g., calculated by <a href="#">svdNorm</a> ).
item	item number for coefficient estimation.
startvals	start values for function minimization.
k	order of monotonic polynomial = $2k+1$ (see Liang & Browne, 2015).

**Value**

b	Vector of polynomial coefficients.
FHAT	Function value at convergence.
counts	Number of function evaluations during minimization (see <a href="#">optim</a> documentation for further details).
AIC	Pseudo scaled Akaike Information Criterion (AIC). Candidate models that produce the smallest AIC suggest the optimal number of parameters given the sample size. Scaling is accomplished by dividing the non-scaled AIC by sample size.
BIC	Pseudo scaled Bayesian Information Criterion (BIC). Candidate models that produce the smallest BIC suggest the optimal number of parameters given the sample size. Scaling is accomplished by dividing the non-scaled BIC by sample size.
convergence	Convergence = 0 indicates that the optimization algorithm converged; convergence=1 indicates that the optimization failed to converge.
.	.

**Author(s)**

Niels Waller

**References**

Liang, L. & Browne, M. W. (2015). A quasi-parametric method for fitting flexible item response functions. *Journal of Educational and Behavioral Statistics*, 40, 5–34.

**Examples**

```
## Not run:
NSubjects <- 2000

## generate sample k=1 FMP data
b <- matrix(c(
  #b0    b1    b2    b3    b4    b5 b6 b7 k
  1.675, 1.974, -0.068, 0.053, 0, 0, 0, 0, 1,
  1.550, 1.805, -0.230, 0.032, 0, 0, 0, 0, 1,
  1.282, 1.063, -0.103, 0.003, 0, 0, 0, 0, 1,
  0.704, 1.376, -0.107, 0.040, 0, 0, 0, 0, 1,
```



```

1.417, 1.413, 0.021, 0.000, 0, 0, 0, 0, 1,
-0.008, 1.349, -0.195, 0.144, 0, 0, 0, 0, 1,
0.512, 1.538, -0.089, 0.082, 0, 0, 0, 0, 1,
0.122, 0.601, -0.082, 0.119, 0, 0, 0, 0, 1,
1.801, 1.211, 0.015, 0.000, 0, 0, 0, 0, 1,
-0.207, 1.191, 0.066, 0.033, 0, 0, 0, 0, 1,
-0.215, 1.291, -0.087, 0.029, 0, 0, 0, 0, 1,
0.259, 0.875, 0.177, 0.072, 0, 0, 0, 0, 1,
-0.423, 0.942, 0.064, 0.094, 0, 0, 0, 0, 1,
0.113, 0.795, 0.124, 0.110, 0, 0, 0, 0, 1,
1.030, 1.525, 0.200, 0.076, 0, 0, 0, 0, 1,
0.140, 1.209, 0.082, 0.148, 0, 0, 0, 0, 1,
0.429, 1.480, -0.008, 0.061, 0, 0, 0, 0, 1,
0.089, 0.785, -0.065, 0.018, 0, 0, 0, 0, 1,
-0.516, 1.013, 0.016, 0.023, 0, 0, 0, 0, 1,
0.143, 1.315, -0.011, 0.136, 0, 0, 0, 0, 1,
0.347, 0.733, -0.121, 0.041, 0, 0, 0, 0, 1,
-0.074, 0.869, 0.013, 0.026, 0, 0, 0, 0, 1,
0.630, 1.484, -0.001, 0.000, 0, 0, 0, 0, 1),
nrow=23, ncol=9, byrow=TRUE)

# generate data using the above item parameters
ex1.data<-genFMPData(NSubj = NSubjects, bParams = b, seed = 345)$data

NItems <- ncol(ex1.data)

# compute (initial) surrogate theta values from
# the normed left singular vector of the centered
# data matrix
thetaInit <- svdNorm(ex1.data)

# Choose model
k <- 1 # order of polynomial = 2k+1

# Initialize matrices to hold output
if(k == 0) {
  startVals <- c(1.5, 1.5)
  bmat <- matrix(0,NItems,6)
  colnames(bmat) <- c(paste("b", 0:1, sep = ""), "FHAT", "AIC", "BIC", "convergence")
}

if(k == 1) {
  startVals <- c(1.5, 1.5, .10, .10)
  bmat <- matrix(0,NItems,8)
  colnames(bmat) <- c(paste("b", 0:3, sep = ""), "FHAT", "AIC", "BIC", "convergence")
}

if(k == 2) {
  startVals <- c(1.5, 1.5, .10, .10, .10, .10)
  bmat <- matrix(0,NItems,10)
  colnames(bmat) <- c(paste("b", 0:5, sep = ""), "FHAT", "AIC", "BIC", "convergence")
}

if(k == 3) {
  startVals <- c(1.5, 1.5, .10, .10, .10, .10, .10, .10)
  bmat <- matrix(0,NItems,12)
  colnames(bmat) <- c(paste("b", 0:7, sep = ""), "FHAT", "AIC", "BIC", "convergence")
}

```

```

}

# estimate item parameters and fit statistics
for(i in 1:NItems){
  out<-FUP(data = ex1.data,thetaInit = thetaInit, item = i, startvals = startVals, k = k)
  Nb <- length(out$b)
  bmat[i,1:Nb] <- out$b
  bmat[i,Nb+1] <- out$FHAT
  bmat[i,Nb+2] <- out$AIC
  bmat[i,Nb+3] <- out$BIC
  bmat[i,Nb+4] <- out$convergence
}

# print results
print(bmat)

## End(Not run)

```

---

gen4PMDData

---

*Generate item response data for 1, 2, 3, or 4-parameter IRT models*


---

## Description

Generate item response data for or 1, 2, 3 or 4-parameter IRT Models.

## Usage

```

gen4PMDData(NSubj, abcdParams, D = 1.702, seed = NULL,
            theta = NULL, thetaMN = 0, thetaVar = 1)

```

## Arguments

NSubj	the desired number of subject response vectors.
abcdParams	a p(items)-by-4 matrix of IRT item parameters: a = discrimination, b = difficulty, c = lower asymptote, and d = upper asymptote.
D	Scaling constant to place the IRF on the normal ogive or logistic metric. Default = 1.702 (normal ogive metric)
seed	Optional seed for the random number generator.
theta	Optional vector of latent trait scores. If theta = NULL (the default value) then gen4PMDData will simulate theta from a normal distribution.
thetaMN	Mean of simulated theta distribution. Default = 0.
thetaVar	Variance of simulated theta distribution. Default = 1

## Value

data	N(subject)-by-p(items) matrix of item response data.
theta	Latent trait scores.
seed	Value of the random number seed.

**Author(s)**

Niels Waller

**Examples**

```
## Generate simulated 4PM data for 2,000 subjects
# 4PM Item parameters from MMPI-A CYN scale

Params<-matrix(c(1.41, -0.79, .01, .98, #1
                1.19, -0.81, .02, .96, #2
                0.79, -1.11, .05, .94, #3
                0.94, -0.53, .02, .93, #4
                0.90, -1.02, .04, .95, #5
                1.00, -0.21, .02, .84, #6
                1.05, -0.27, .02, .97, #7
                0.90, -0.75, .04, .73, #8
                0.80, -1.42, .06, .98, #9
                0.71,  0.13, .05, .94, #10
                1.01, -0.14, .02, .81, #11
                0.63,  0.18, .18, .97, #12
                0.68,  0.18, .02, .87, #13
                0.60, -0.14, .09, .96, #14
                0.85, -0.71, .04, .99, #15
                0.83, -0.07, .05, .97, #16
                0.86, -0.36, .03, .95, #17
                0.66, -0.64, .04, .77, #18
                0.60,  0.52, .04, .94, #19
                0.90, -0.06, .02, .96, #20
                0.62, -0.47, .05, .86, #21
                0.57,  0.13, .06, .93, #22
                0.77, -0.43, .04, .97),23,4, byrow=TRUE)

data <- gen4PMDData(NSubj=2000, abcdParams = Params, D = 1.702,
                    seed = 123, thetaMN = 0, thetaVar = 1)$data

cat("\nClassical item difficulties for simulated data")
print( round( apply(data,2,mean),2) )
```

genCorr

*Generate Correlation Matrices with User-Defined Eigenvalues***Description**

Uses the Marsaglia and Olkin (1984) algorithm to generate correlation matrices with user-defined eigenvalues.

**Usage**

```
genCorr(eigenval, seed='rand')
```

**Arguments**

eigenval	A vector of eigenvalues that must sum to the order of the desired correlation matrix. For example: if you want a correlation matrix of order 4, then you need 4 eigenvalues that sum to 4. A warning message will display if <code>sum(eigenval) != length(eigenval)</code>
seed	Either a user supplied seed for the random number generator or 'rand' for a function generated seed. Default seed='rand'.

**Value**

Returns a correlation matrix with the eigen-structure specified by eigenval.

**Author(s)**

Jeff Jones

**References**

- Jones, J. A. (2010). GenCorr: An R routine to generate correlation matrices from a user-defined eigenvalue structure. *Applied Psychological Measurement*, 34, 68-69.
- Marsaglia, G., & Olkin, I. (1984). Generating correlation matrices. *SIAM J. Sci. and Stat. Comput.*, 5, 470-475.

**Examples**

```
## Example
## Generate a correlation matrix with user-specified eigenvalues
set.seed(123)
R <- genCorr(c(2.5, 1, 1, .3, .2))

print(round(R, 2))

#>      [,1] [,2] [,3] [,4] [,5]
#> [1,]  1.00  0.08 -0.07 -0.07  0.00
#> [2,]  0.08  1.00  0.00 -0.60  0.53
#> [3,] -0.07  0.00  1.00  0.51 -0.45
#> [4,] -0.07 -0.60  0.51  1.00 -0.75
#> [5,]  0.00  0.53 -0.45 -0.75  1.00

print(eigen(R)$values)

#[1] 2.5 1.0 1.0 0.3 0.2
```

---

genFMPData

*Generate item response data for a filtered monotonic polynomial IRT model*

---

**Description**

Generate item response data for the filtered polynomial IRT model.

**Usage**

```
genFMPData(NSubj, bParams, theta = NULL, thetaMN = 0, thetaVar = 1, seed)
```

**Arguments**

<code>NSubj</code>	the desired number of subject response vectors.
<code>bParams</code>	a p(items)-by-9 matrix of polynomial coefficients and model designations. Columns 1 - 8 hold the polynomial coefficients; column 9 holds the value of k.
<code>theta</code>	A user-supplied vector of latent trait scores. Default theta = NULL.
<code>thetaMN</code>	If theta = NULL genFMPdata will simulate random normal deviates from a population with mean thetaMN and variance thetaVar.
<code>thetaVar</code>	If theta = NULL genFMPData will simulate random normal deviates from a population with mean thetaMN and variance thetaVar.
<code>seed</code>	initial seed for the random number generator.

**Value**

<code>theta</code>	theta values used for data generation
<code>data</code>	N(subject)-by-p(items) matrix of item response data.
<code>seed</code>	Value of the random number seed.

**Author(s)**

Niels Waller

**Examples**

```
# The following code illustrates data generation for
# an FMP of order 3 (i.e., 2k+1)

# data will be generated for 2000 examinees
NSubjects <- 2000

## Example item paramters, k=1 FMP
b <- matrix(c(
  #b0    b1    b2    b3    b4    b5 b6 b7 k
  1.675, 1.974, -0.068, 0.053, 0, 0, 0, 0, 1,
  1.550, 1.805, -0.230, 0.032, 0, 0, 0, 0, 1,
  1.282, 1.063, -0.103, 0.003, 0, 0, 0, 0, 1,
  0.704, 1.376, -0.107, 0.040, 0, 0, 0, 0, 1,
  1.417, 1.413, 0.021, 0.000, 0, 0, 0, 0, 1,
  -0.008, 1.349, -0.195, 0.144, 0, 0, 0, 0, 1,
  0.512, 1.538, -0.089, 0.082, 0, 0, 0, 0, 1,
  0.122, 0.601, -0.082, 0.119, 0, 0, 0, 0, 1,
  1.801, 1.211, 0.015, 0.000, 0, 0, 0, 0, 1,
  -0.207, 1.191, 0.066, 0.033, 0, 0, 0, 0, 1,
  -0.215, 1.291, -0.087, 0.029, 0, 0, 0, 0, 1,
  0.259, 0.875, 0.177, 0.072, 0, 0, 0, 0, 1,
  -0.423, 0.942, 0.064, 0.094, 0, 0, 0, 0, 1,
  0.113, 0.795, 0.124, 0.110, 0, 0, 0, 0, 1,
  1.030, 1.525, 0.200, 0.076, 0, 0, 0, 0, 1,
```

```

0.140, 1.209, 0.082, 0.148, 0, 0, 0, 0, 1,
0.429, 1.480, -0.008, 0.061, 0, 0, 0, 0, 1,
0.089, 0.785, -0.065, 0.018, 0, 0, 0, 0, 1,
-0.516, 1.013, 0.016, 0.023, 0, 0, 0, 0, 1,
0.143, 1.315, -0.011, 0.136, 0, 0, 0, 0, 1,
0.347, 0.733, -0.121, 0.041, 0, 0, 0, 0, 1,
-0.074, 0.869, 0.013, 0.026, 0, 0, 0, 0, 1,
0.630, 1.484, -0.001, 0.000, 0, 0, 0, 0, 1),
nrow=23, ncol=9, byrow=TRUE)

# generate data using the above item paramters
data<-genFMPData(NSubj = NSubjects, bParams=b, seed=345)$data

```

HS9Var

*9 Variables from the Holzinger and Swineford (1939) Dataset***Description**

Mental abilities data on seventh- and eighth-grade children from the classic Holzinger and Swineford (1939) dataset.

**Usage**

```
data("HS9Var")
```

**Format**

A data frame with 301 observations on the following 15 variables.

**id** subject identifier  
**sex** gender  
**ageyr** age, year part  
**agemo** age, month part  
**school** school name (Pasteur or Grant-White)  
**grade** grade  
**x1** Visual perception  
**x2** Cubes  
**x3** Lozenges  
**x4** Paragraph comprehension  
**x5** Sentence completion  
**x6** Word meaning  
**x7** Speeded addition  
**x8** Speeded counting of dots  
**x9** Speeded discrimination straight and curved capitals

**Source**

These data were retrieved from the lavaan package. The complete data for all 26 tests are available in the MBESS package.

## References

- Holzinger, K., and Swineford, F. (1939). A study in factor analysis: The stability of a bifactor solution. Supplementary Educational Monograph, no. 48. Chicago: University of Chicago Press.
- Joreskog, K. G. (1969). A general approach to confirmatory maximum likelihood factor analysis. *Psychometrika*, 34, 183-202.

## Examples

```
data(HS9Var)
head(HS9Var)
```

---

irf	<i>Plot item response functions for polynomial IRT models.</i>
-----	--

---

## Description

Plot model-implied (and possibly empirical) item response function for polynomial IRT models.

## Usage

```
irf(data, bParams, item, plotERF = TRUE, thetaEAP = NULL,
    minCut = -3, maxCut = 3, NCuts = 9)
```

## Arguments

data	N(subjects)-by-p(items) matrix of 0/1 item response data.
bParams	p(items)-by-9 matrix. The first 8 columns of the matrix should contain the FMP or FUP polynomial coefficients for the p items. The 9th column contains the value of k for each item (where the item specific order of the polynomial is $2k+1$ ).
item	The IRF for item will be plotted.
plotERF	A logical that determines whether to plot discrete values of the empirical response function.
thetaEAP	If plotERF=TRUE, the user must supply previously calculated eap trait estimates to thetaEAP.
minCut, maxCut	If plotERF=TRUE, the program will (attempt to) plot NCuts points of the empirical response function between trait values of minCut and maxCut Default minCut = -3. Default maxCut = 3.
NCuts	Desired number of bins for the empirical response function.

## Author(s)

Niels Waller

**Examples**

```

NSubjects <- 2000
NItems <- 15

itmParameters <- matrix(c(
# b0 b1 b2 b3 b4 b5, b6, b7, k
-1.05, 1.63, 0.00, 0.00, 0.00, 0, 0, 0, 0, #1
-1.97, 1.75, 0.00, 0.00, 0.00, 0, 0, 0, 0, #2
-1.77, 1.82, 0.00, 0.00, 0.00, 0, 0, 0, 0, #3
-4.76, 2.67, 0.00, 0.00, 0.00, 0, 0, 0, 0, #4
-2.15, 1.93, 0.00, 0.00, 0.00, 0, 0, 0, 0, #5
-1.25, 1.17, -0.25, 0.12, 0.00, 0, 0, 0, 1, #6
1.65, 0.01, 0.02, 0.03, 0.00, 0, 0, 0, 1, #7
-2.99, 1.64, 0.17, 0.03, 0.00, 0, 0, 0, 1, #8
-3.22, 2.40, -0.12, 0.10, 0.00, 0, 0, 0, 1, #9
-0.75, 1.09, -0.39, 0.31, 0.00, 0, 0, 0, 1, #10
-1.21, 9.07, 1.20, -0.01, -0.01, 0.01, 0, 0, 2, #11
-1.92, 1.55, -0.17, 0.50, -0.01, 0.01, 0, 0, 2, #12
-1.76, 1.29, -0.13, 1.60, -0.01, 0.01, 0, 0, 2, #13
-2.32, 1.40, 0.55, 0.05, -0.01, 0.01, 0, 0, 2, #14
-1.24, 2.48, -0.65, 0.60, -0.01, 0.01, 0, 0, 2), #15
15, 9, byrow=TRUE)

ex1.data <- genFMPData(NSubj = NSubjects, bParams = itmParameters,
                      seed = 345)$data

## compute initial theta surrogates
thetaInit <- svdNorm(ex1.data)

## For convenience we assume that the item parameter
## estimates equal their population values. In practice,
## item parameters would be estimated at this step.
itmEstimates <- itmParameters

## calculate eap estimates for mixed models
thetaEAP <- eap(data = ex1.data, bParams = itmEstimates, NQuad = 21,
               priorVar = 2,
               mintheta = -4, maxtheta = 4)

## plot irf and erf for item 1
irf(data = ex1.data, bParams = itmEstimates,
    item = 1,
    plotERF = TRUE,
    thetaEAP)

## plot irf and erf for item 12
irf(data = ex1.data, bParams = itmEstimates,
    item = 12,
    plotERF = TRUE,
    thetaEAP)

```



**Description**

Compute basic descriptives for binary item analysis

**Usage**

```
itemDescriptives(X, digits=3)
```

**Arguments**

**X** a matrix of binary (0/1) item responses.  
**digits** number of digits to print.

**Value**

**alpha** Coefficient alpha for the total scale.  
**means** item means.  
**standard deviations** item standard deviations.  
**pt. biserial correlations** corrected item-total point biserial correlations.  
**biserial correlations** corrected item-total point biserial correlations.  
**corrected.alpha** corrected (leave item out) alpha coefficients.

**Author(s)**

Niels Waller

**Examples**

```
## Example 1: generating binary data to match
## an existing binary data matrix
##
## Generate correlated scores using factor
## analysis model
## X <- Z * L' + U * D
## Z is a vector of factor scores
## L is a factor loading matrix
## U is a matrix of unique factor scores
## D is a scaling matrix for U

Nsubj <- 2000
L <- matrix( rep(.707,5), nrow = 5, ncol = 1)
Z <- as.matrix(rnorm(Nsubj))
U <- matrix(rnorm(Nsubj * 5), nrow = Nsubj, ncol = 5)
tmp <- sqrt(1 - L^2)
D <- matrix(0, 5, 5)
diag(D) <- tmp
X <- Z %*% t(L) + U %*% D

cat("\nCorrelation of continuous scores\n")
print(round(cor(X),3))
```

```

thresholds <- c(.2,.3,.4,.5,.6)

Binary<-matrix(0,Nsubj,5)
for(i in 1:5){
  Binary[X[,i]<=thresholds[i],i]<-1
}

cat("\nCorrelation of Binary scores\n")
print(round(cor(Binary),3))

## Now use 'bigen' to generate binary data matrix with
## same correlations as in Binary

z <- bigen(data = Binary, n = 5000)

cat("\n\nnames in returned object\n")
print(names(z))

cat("\nCorrelation of Simulated binary scores\n")
print(round( cor(z$data), 3))

cat("Observed thresholds of simulated data:\n")
cat( apply(z$data, 2, mean) )

itemDescriptives(z$data)

```

---

kurt

---

*Calculate Univariate Kurtosis for a Vector or Matrix*


---

### Description

Calculate univariate kurtosis for a vector or matrix (algorithm G2 in Joanes & Gill, 1998).

### Usage

```
kurt(x)
```

### Arguments

x Either a vector or matrix of numeric values.

### Value

Kurtosis for each column in x.

### Author(s)

Niels Waller

### References

Joanes, D. N. & Gill, C. A. (1998). Comparing measures of sample skewness and kurtosis. *The Statistician*, 47, 183-189.

**See Also**[skew](#)**Examples**

```
x <- matrix(rnorm(1000), 100, 10)
print(kurt(x))
```

monte

*Simulate Clustered Data with User-Defined Properties***Description**

Function for simulating clustered data with user defined characteristics such as: within cluster indicator correlations, within cluster indicator skewness values, within cluster indicator kurtosis values, and cluster separations as indexed by each variable (indicator validities).

**Usage**

```
monte(seed, nvar, nclus, clus.size, eta2, cor.list, random.cor,
      skew.list, kurt.list, secor, compactness, sortMeans)
```

**Arguments**

seed	Required: An integer to be used as the random number seed.
nvar	Required: Number of variables to simulate.
nclus	Required: Number of clusters to simulate. <i>Note</i> that number of clusters must be equal to or greater than 2.
clus.size	Required: Number of objects in each cluster.
eta2	Required: A vector of indicator validities that range from 0 to 1. Higher numbers produce clusters with greater separation on that indicator.
cor.list	Optional: A list of correlation matrices. There should be one correlation matrix for each cluster. The first correlation matrix will represent the indicator correlations within cluster 1. The second correlation matrix will represent the indicator correlations for cluster 2. Etc.
random.cor	Optional: Set to TRUE to generate a common within cluster correlation matrix.
skew.list	Optional: A list of within cluster indicator skewness values.
kurt.list	Optional: A list of within cluster indicator kurtosis values.
secor	Optional: If 'random.cor = TRUE' then 'secor' determines the standard error of the simulated within group correlation matrices.
compactness	Optional: A vector of cluster compactness parameters. The meaning of this option is explained Waller et al. (1999). Basically, 'compactness' allows users some control over cluster overlap without changing indicator validities. See the example below for an illustration.
sortMeans	Optional: A logical that determines whether the latent means will be sorted by taxon. Default = TRUE

**Value**

data	The simulated data. The 1st column of 'data' denotes cluster membership.
lmn	The cluster indicator means.
fl	The factor loading matrix as described in Waller, et al. 1999.
fs	The unique values of the linearized factor scores.
call	The call.
nclus	Number of clusters.
nvar	Number of variables.
cor.list	The input within cluster correlation matrices.
skew.list	The input within cluster indicator skewness values.
kurt.list	The input within cluster indicator kurtosis values.
clus.size	The number of observations in each cluster.
eta2	Vector of indicator validities.
seed	The random number seed.

**Author(s)**

Niels Waller

**References**

- Fleishman, A. I (1978). A method for simulating non-normal distributions. *Psychometrika*, 43, 521-532.
- Vale, D. C., & Maurelli, V. A. (1983). Simulating multivariate nonnormal distributions. *Psychometrika*, 48, 465-471.
- Waller, N. G., Underhill, J. M., & Kaiser, H. A. (1999). A method for generating simulated plas-modes and artificial test clusters with user-defined shape, size, and orientation. *Multivariate Behavioral Research*, 34, 123-142.

**Examples**

```
## Example 1
## Simulating Fisher's Iris data
# The original data were reported in:
# Fisher, R. A. (1936) The use of multiple measurements in taxonomic
#   problems. Annals of Eugenics, 7, Part II, 179-188.
#
# This example includes 3 clusters. Each cluster represents
# an Iris species: Setosa, Versicolor, and Virginica.
# On each species, four variables were measured: Sepal Length,
# Sepal Width, Petal Length, and Petal Width.
#
# The within species (cluster) correlations of the flower
# indicators are as follows:
#
# Iris Type 1:
#   [,1] [,2] [,3] [,4]
# [1,] 1.000 0.743 0.267 0.178
# [2,] 0.743 1.000 0.278 0.233
# [3,] 0.267 0.278 1.000 0.332
```

```

# [4,] 0.178 0.233 0.332 1.000
#
# Iris Type 2
#      [,1] [,2] [,3] [,4]
# [1,] 1.000 0.526 0.754 0.546
# [2,] 0.526 1.000 0.561 0.664
# [3,] 0.754 0.561 1.000 0.787
# [4,] 0.546 0.664 0.787 1.000
#
# Iris Type 3
#      [,1] [,2] [,3] [,4]
# [1,] 1.000 0.457 0.864 0.281
# [2,] 0.457 1.000 0.401 0.538
# [3,] 0.864 0.401 1.000 0.322
# [4,] 0.281 0.538 0.322 1.000
#
# 'monte' expects a list of correlation matrices
#

#create a list of within species correlations
data(iris)
cormat <- cm <- lapply(split(iris[,1:4], iris[,5]), cor)

# create a list of within species indicator
# skewness and kurtosis
sk.lst <- list(c(0.120, 0.041, 0.106, 1.254),
               c(0.105, -0.363, -0.607, -0.031),
               c(0.118, 0.366, 0.549, -0.129) )

kt.lst <- list(c(-0.253, 0.955, 1.022, 1.719),
               c(-0.533, -0.366, 0.048, -0.410),
               c(0.033, 0.706, -0.154, -0.602) )

#Generate a new sample of iris data
my.iris <- monte(seed=123, nvar = 4, nclus = 3, cor.list = cormat,
                 clus.size = c(50, 50, 50),
                 eta2=c(0.619, 0.401, 0.941, 0.929),
                 random.cor = FALSE,
                 skew.list = sk.lst,
                 kurt.list = kt.lst,
                 secor = .3, compactness=c(1, 1, 1),
                 sortMeans = TRUE)

summary(my.iris)
plot(my.iris)

# Now generate a new data set with the sample indicator validities
# as before but with different cluster compactness values.

my.iris2<-monte(seed = 123, nvar = 4, nclus = 3,
                 cor.list = cormat, clus.size = c(50, 50, 50),
                 eta2 = c(0.619, 0.401, 0.941, 0.929), random.cor = FALSE,
                 skew.list = sk.lst ,kurt.list = kt.lst,
                 secor = .3,

```

```

compactness=c(2, .5, .5),
sortMeans = TRUE)

summary(my.iris2)

# Notice that cluster 1 has been blow up whereas clusters 2 and 3 have been shrunk.
plot(my.iris2)

### Now compare your original results with the actual
## Fisher iris data
library(lattice)
data(iris)
super.sym <- trellis.par.get("superpose.symbol")
splom(~iris[1:4], groups = Species, data = iris,
      #panel = panel.superpose,
      key = list(title = "Three Varieties of Iris",
                  columns = 3,
                  points = list(pch = super.sym$pch[1:3],
                                col = super.sym$col[1:3]),
                  text = list(c("Setosa", "Versicolor", "Virginica"))))

##### EXAMPLE 2 #####

## Example 2
## Simulating data for Taxometric
## Monte Carlo Studies.
##
## In this four part example we will
## generate two group mixtures
## (Complement and Taxon groups)
## under four conditions.
##
## In all conditions
## base rate (BR) = .20
## 3 indicators
## indicator validities = .50
## (This means that 50 percent of the total
## variance is due to the mixture.)
##
##
## Condition 1:
## All variables have a slight degree
## of skewness (.10) and kurtosis (.10).
## Within group correlations = 0.00.
##
##
## Condition 2:
## In this conditon we generate data in which the
## complement and taxon distributions differ in shape.
## In the complement group all indicators have
## skewness values of 1.75 and kurtosis values of 3.75.
## In the taxon group all indicators have skewness values
## of .50 and kurtosis values of 0.

```

```

## As in the previous condition, all within group
## correlations (nuisance covariance) are 0.00.
##
##
## Conditon 3:
## In this condition we retain all previous
## characteristics except that the within group
## indicator correlations now equal .80
## (they can differ between groups).
##
##
## Conditon 4:
## In this final condition we retain
## all previous data characteristics except that
## the variances of the indicators in the complement
## class are now 5 times the indicator variances
## in the taxon class (while maintaining indicator skewness,
## kurtosis, correlations, etc.).

##-----

library(lattice)

#####
##      Condition 1
#####
in.nvar <- 3  ##Number of variables
in.nclus <-2  ##Number of taxa
in.seed <- 123
BR <- .20     ## Base rate of higher taxon

## Within taxon indicator skew and kurtosis
in.skew.list <- list(c(.1, .1, .1),c(.1, .1, .1))
in.kurt.list <- list(c(.1, .1, .1),c(.1, .1, .1))

## Indicator validities
in.eta2 <- c(.50, .50, .50)

## Groups sizes for Population
BigN <- 100000
in.clus.size <- c(BigN*(1-BR), BR * BigN)

## Generate Population of scores with "monte"
sample.data <- monte(seed = in.seed,
                     nvar=in.nvar,
                     nclus = in.nclus,
                     clus.size = in.clus.size,
                     eta2 = in.eta2,
                     skew.list = in.skew.list,
                     kurt.list = in.kurt.list)

output <- summary(sample.data)

```

```

z <- data.frame(sample.data$data[sample(1:BigN, 600, replace=FALSE),])
z[,2:4] <- scale(z[,2:4])
names(z) <- c("id","v1","v2","v3")

#trellis.device()
trellis.par.set( col.whitebg() )
print(
  cloud(v3 ~ v1 * v2,
    groups = as.factor(id),data=z,
    subpanel = panel.superpose,
    zlim=c(-4, 4),
    xlim=c(-4, 4),
    ylim=c(-4, 4),
    main="",
    screen = list(z = 20, x = -70)),
  position=c(.1, .5, .5, 1), more = TRUE)

#####
##      Condition 2
#####

## Within taxon indicator skew and kurtosis
in.skew.list <- list(c(1.75, 1.75, 1.75),c(.50, .50, .50))
in.kurt.list <- list(c(3.75, 3.75, 3.75),c(0, 0, 0))

## Generate Population of scores with "monte"
sample.data <- monte(seed = in.seed,
  nvar = in.nvar,
  nclus = in.nclus,
  clus.size = in.clus.size,
  eta2 = in.eta2,
  skew.list = in.skew.list,
  kurt.list = in.kurt.list)

output <- summary(sample.data)

z <- data.frame(sample.data$data[sample(1:BigN, 600, replace=FALSE),])
z[,2:4] <- scale(z[, 2:4])
names(z) <-c("id", "v1","v2", "v3")

print(
  cloud(v3 ~ v1 * v2,
    groups = as.factor(id), data = z,
    subpanel = panel.superpose,
    zlim = c(-4, 4),
    xlim = c(-4, 4),
    ylim = c(-4, 4),
    main="",
    screen = list(z = 20, x = -70)),
  position = c(.5, .5, 1, 1), more = TRUE)

#####
##      Condition 3

```



```
#####

## Set within group correlations to .80
cormat <- matrix(.80, 3, 3)
diag(cormat) <- rep(1, 3)
in.cor.list <- list(cormat, cormat)

## Generate Population of scores with "monte"
sample.data <- monte(seed = in.seed,
                     nvar = in.nvar,
                     nclus = in.nclus,
                     clus.size = in.clus.size,
                     eta2 = in.eta2,
                     skew.list = in.skew.list,
                     kurt.list = in.kurt.list,
                     cor.list = in.cor.list)

output <- summary(sample.data)

z <- data.frame(sample.data$data[sample(1:BigN, 600,
                                       replace = FALSE), ])
z[,2:4] <- scale(z[, 2:4])
names(z) <- c("id", "v1", "v2", "v3")

##trellis.device()
##trellis.par.set( col.whitebg() )
print(
  cloud(v3 ~ v1 * v2,
        groups = as.factor(id),data=z,
        subpanel = panel.superpose,
        zlim = c(-4, 4),
        xlim = c(-4, 4),
        ylim = c(-4, 4),
        main="",
        screen = list(z = 20, x = -70)),
  position = c(.1, .0, .5, .5), more = TRUE)

#####
##      Condition 4
#####

## Change compactness so that variance of
## complement indicators is 5 times
## greater than variance of taxon indicators

v <- ( 2 * sqrt(5))/(1 + sqrt(5))
in.compactness <- c(v, 2-v)

## Generate Population of scores with "monte"
sample.data <- monte(seed = in.seed,
                     nvar = in.nvar,
                     nclus = in.nclus,
                     clus.size = in.clus.size,
                     eta2 = in.eta2,
                     skew.list = in.skew.list,
                     kurt.list = in.kurt.list,
```

```

cor.list = in.cor.list,
compactness = in.compactness)

output <- summary(sample.data)

z <- data.frame(sample.data$data[sample(1:BigN, 600, replace = FALSE), ])
z[, 2:4] <- scale(z[, 2:4])
names(z) <- c("id", "v1", "v2", "v3")
print(
  cloud(v3 ~ v1 * v2,
    groups = as.factor(id), data=z,
    subpanel = panel.superpose,
    zlim = c(-4, 4),
    xlim = c(-4, 4),
    ylim = c(-4, 4),
    main="",
    screen = list(z = 20, x = -70)),
  position = c(.5, .0, 1, .5), more = TRUE)

```

---

monte1	<i>Simulate Multivariate Non-normal Data by Vale &amp; Maurelli (1983)</i> <i>Method</i>
--------	---

---

## Description

Function for simulating multivariate nonnormal data by the methods described by Fleishman (1978) and Vale & Maurelli (1983).

## Usage

```
monte1(seed, nvar, nsub, cormat, skewvec, kurtvec)
```

## Arguments

seed	An integer to be used as the random number seed.
nvar	Number of variables to simulate.
nsub	Number of simulated subjects (response vectors).
cormat	The desired correlation matrix.
skewvec	A vector of indicator skewness values.
kurtvec	A vector of indicator kurtosis values.

## Value

data	The simulated data.
call	The call.
nsub	Number of subjects.
nvar	Number of variables.
cormat	The desired correlation matrix.
skewvec	The desired indicator skewness values.
kurtvec	The desired indicator kurtosis values.
seed	The random number seed.

**Author(s)**

Niels Waller

**References**

Fleishman, A. I (1978). A method for simulating non-normal distributions. *Psychometrika*, 43, 521-532.

Vale, D. C., & Maurelli, V. A. (1983). Simulating multivariate nonnormal distributions. *Psychometrika*, 48, 465-471.

**See Also**

[monte](#), [summary.monte](#), [summary.monte1](#)

**Examples**

```
## Generate dimensional data for 4 variables.
## All correlations = .60; all variable
## skewness = 1.75;
## all variable kurtosis = 3.75

cormat <- matrix(.60,4,4)
diag(cormat) <- 1

nontaxon.dat <- monte1(seed = 123, nsub = 100000, nvar = 4, skewvec = rep(1.75, 4),
                      kurtvec = rep(3.75, 4), cormat = cormat)

print(cor(nontaxon.dat$data), digits = 3)
print(apply(nontaxon.dat$data, 2, skew), digits = 3)
print(apply(nontaxon.dat$data, 2, kurt), digits = 3)
```

---

normalCor

---

*Compute Normal-Theory Covariances for Correlations*


---

**Description**

Compute normal-theory covariances for correlations

**Usage**

```
normalCor(R, Nobs)
```

**Arguments**

R                      a p x p matrix of correlations.  
Nobs                    Number of observations.

**Value**

A normal-theory covariance matrix of correlations.

**Author(s)**

Jeff Jones and Niels Waller

**References**

Nel, D.G. (1985). A matrix derivation of the asymptotic covariance matrix of sample correlation coefficients. *Linear algebra and its applications*, 67, 137–145.

**See Also**

[adfCor](#)

**Examples**

```
data(Harman23.cor)
normalCor(Harman23.cor$cov, Nobs = 305)
```

---

normF	<i>Compute the Frobenius norm of a matrix</i>
-------	---

---

**Description**

A function to compute the Frobenius norm of a matrix

**Usage**

```
normF(X)
```

**Arguments**

X                      A matrix.

**Value**

The Frobenius norm of X.

**Author(s)**

Niels Waller

**Examples**

```
data(BadRLG)
out <- smoothLG(R = BadRLG, Penalty = 50000)
cat("\nGradient at solution:", out$gr, "\n")
cat("\nNearest Correlation Matrix\n")
print( round(out$RLG,8) )
cat("\nFrobenius norm of (NPD - PSD) matrix\n")
print(normF(BadRLG - out$RLG ))
```

---

plot.monte	<i>Plot Method for Class Monte</i>
------------	------------------------------------

---

**Description**

plot method for class "monte"

**Usage**

```
## S3 method for class 'monte'
plot(x, ...)
```

**Arguments**

x	An object of class 'monte', usually, a result of a call to monte.
...	Optional arguments passed to plotting function.

**Value**

The function `plot.monte` creates a scatter plot of matrices plot (a splom plot). Cluster membership is denoted by different colors in the plot.

**Examples**

```
#plot(monte.object)
```

---

r2d	<i>Convert Radians to Degrees</i>
-----	-----------------------------------

---

**Description**

Convert radian measure to degrees.

**Usage**

```
r2d(radian)
```

**Arguments**

radian	Radian measure of an angle
--------	----------------------------

**Value**

Degree measure of an angle

**Examples**

```
r2d(.5*pi)
```

rarc

*Rotate Points on the Surface on an N-Dimensional Ellipsoid***Description**

Rotate between two points on the surface on an n-dimensional ellipsoid. The hyper-ellipsoid is composed of all points, B, such that  $B' R_{xx} B = R_{sq}$ . Vector B contains standardized regression coefficients.

**Usage**

```
rarc(Rxx, Rsq, b1, b2, Npoints)
```

**Arguments**

Rxx	Predictor correlation matrix.
Rsq	Model coefficient of determination.
b1	First point on ellipsoid. If b1 and b2 are scalars then choose scaled eigenvectors v[b1] and v[b2] as the start and end vectors.
b2	Second point on ellipsoid. If b1 and b2 are scalars then choose scaled eigenvectors v[b1] and v[b2] as the start and end vectors.
Npoints	Generate “Npoints” +1 OLS coefficient vectors between b1 and b2.

**Value**

b	N+1 sets of OLS coefficient vectors between b1 and b2.
---	--

**Author(s)**

Niels Waller and Jeff Jones.

**References**

Waller, N. G. & Jones, J. A. (2011). Investigating the performance of alternate regression weights by studying all possible criteria in regression models with a fixed set of predictors. *Psychometrika*, 76, 410-439.

**Examples**

```
## Example
## GRE/GPA Data
##-----##
R <- Rxx <- matrix(c(1.00, .56, .77,
                    .56, 1.00, .73,
                    .77, .73, 1.00), 3, 3)

## GPA validity correlations
rxy <- c(.39, .34, .38)
b <- solve(Rxx) %*% rxy

Rsqr <- t(b) %*% Rxx %*% b
```

```

N <- 200

b <- rarc(Rxx = R, Rsq, b1 = 1, b2 = 3, Npoints = N)

## compute validity vectors
r <- Rxx %%% b
N <- N + 1
Rsqr <- Rsq.unit <- rep(0, N)

for(i in 1:N){
  ## eval performance of unit weights
  Rsq.unit[i] <- (t(sign(r[,i])) %%% r[,i])^2 /
    (t(sign(r[,i])) %%% R %%% sign(r[,i]))

  ## eval performance of correlation weights
  Rsqr[i] <- (t(r[,i]) %%% r[,i])^2 / (t(r[,i]) %%% R %%% r[,i])
}

cat("\nAverage relative performance of unit weights across elliptical arc:",
    round(mean(Rsq.unit)/Rsq,3) )
cat("\n\nAverage relative performance of r weights across elliptical arc:",
    round(mean(Rsqr)/Rsq,3) )

plot(seq(0, 90, length = N), Rsqr, typ = "l",
      ylim = c(0, .20),
      xlim = c(0, 95),
      lwd = 3,
      ylab = expression(R^2),
      xlab = expression(paste("Degrees from ",b[1]," in the direction of ",b[2])),
      cex.lab = 1.25, lab = c(10, 5, 5))
points(seq(0, 90, length = N), Rsq.unit,
        type = "l",
        lty = 2, lwd = 3)
legend(x = 0,y = .12,
       legend = c("r weights", "unit weights"),
       lty = c(1, 2),
       lwd = c(4, 3),
       cex = 1.5)

```

rcone

*Generate a Cone of Regression Coefficient Vectors***Description**

Compute a cone of regression vectors with a constant R-squared around a target vector.

**Usage**

```
rcone(R,Rsq,b,axis1,axis2,deg,Npoints=360)
```

**Arguments**

R                      Predictor correlation matrix.

Rsq	Coefficient of determination.
b	Target vector of OLS regression coefficients.
axis1	1st axis of rotation plane.
axis2	2nd axis of rotation plane.
deg	All vectors b.i will be 'deg' degrees from b.
Npoints	Number of rotation vectors, default = 360.

**Value**

b.i	Npoints values of b.i
-----	-----------------------

**Author(s)**

Niels Waller and Jeff Jones

**References**

Waller, N. G. & Jones, J. A. (2011). Investigating the performance of alternate regression weights by studying all possible criteria in regression models with a fixed set of predictors. *Psychometrika*, 76, 410-439.

**Examples**

```
R <- matrix(.5, 4, 4)
diag(R) <- 1

Npoints <- 1000
Rsq <- .40
NumDeg <- 20
V <- eigen(R)$vectors

## create b parallel to v[,3]
## rotate in the 2 - 4 plane
b <- V[,3]
bsq <- t(b) %*% R %*% b
b <- b * sqrt(Rsq/bsq)
b.i <- rcone(R, Rsq, b, V[,2], V[,4], deg = NumDeg, Npoints)

t(b.i[,1]) %*% R %*% b.i[,1]
t(b.i[,25]) %*% R %*% b.i[,25]
```

---

rcor

---

*Generate Random PSD Correlation Matrices*


---

**Description**

Generate random PSD correlation matrices.

**Usage**

```
rcor(Nvar)
```



**Arguments**

Nvar                      An integer that determines the order of the random correlation matrix.

**Details**

rcor generates random PSD correlation matrices by (1) generating Nvar squared random normal deviates, (2) scaling the deviates to sum to Nvar, and then (3) placing the scaled values into a diagonal matrix L. Next, (4) an Nvar x Nvar orthogonal matrix, Q, is created by performing a QR decomposition of a matrix, M, that contains random normal deviates. (5) A PSD covariance matrix, C, is created from  $Q L Q^T$  and then (6) scaled to a correlation metric.

**Value**

A random correlation matrix.

**Author(s)**

Niels Waller

**See Also**

[genCorr](#)

**Examples**

```
R <- rcor(4)
print( R )
```

---

rellipsoid	<i>Generate Uniformly Spaced OLS Regression Coefficients that Yield a User-Supplied R-Squared Value</i>
------------	---

---

**Description**

Given predictor matrix R, generate OLS regression coefficients that yield a user-supplied R-Squared value. These regression coefficient vectors will be uniformly spaced on the surface of a (hyper) ellipsoid.

**Usage**

```
rellipsoid(R, Rsq, Npoints)
```

**Arguments**

R                      A p x p predictor correlation matrix.  
 Rsq                    A user-supplied R-squared value.  
 Npoints                Desired number of generated regression vectors.

**Value**

b                      A p x Npoints matrix of regression coefficients

**Author(s)**

Niels Waller and Jeff Jones.

**References**

Waller, N. G. and Jones, J. A. (2011). Investigating the performance of alternate regression weights by studying all possible criteria in regression models with a fixed set of predictors. *Psychometrika*, 76, 410-439.

**Examples**

```
## generate uniformly distributed regression vectors
## on the surface of a 14-dimensional ellipsoid
N <- 10000
Rsqr <- .21

# Correlations from page 224 WAIS-III manual
# The Psychological Corporation (1997).
wais3 <- matrix(
  c(1, .76, .58, .43, .75, .75, .42, .54, .41, .57, .64, .54, .50, .53,
    .76, 1, .57, .36, .69, .71, .45, .52, .36, .63, .68, .51, .47, .54,
    .58, .57, 1, .45, .65, .60, .47, .48, .43, .59, .60, .49, .56, .47,
    .43, .36, .45, 1, .37, .40, .60, .30, .32, .34, .35, .28, .35, .29,
    .75, .69, .65, .37, 1, .70, .44, .54, .34, .59, .62, .54, .45, .50,
    .75, .71, .60, .40, .70, 1, .42, .51, .44, .53, .60, .50, .52, .44,
    .42, .45, .47, .60, .44, .42, 1, .46, .49, .47, .43, .27, .50, .42,
    .54, .52, .48, .30, .54, .51, .46, 1, .45, .50, .58, .55, .53, .56,
    .41, .36, .43, .32, .34, .44, .49, .45, 1, .47, .49, .41, .70, .38,
    .57, .63, .59, .34, .59, .53, .47, .50, .47, 1, .63, .62, .58, .66,
    .64, .68, .60, .35, .62, .60, .43, .58, .49, .63, 1, .59, .50, .59,
    .54, .51, .49, .28, .54, .50, .27, .55, .41, .62, .59, 1, .48, .53,
    .50, .47, .56, .35, .45, .52, .50, .53, .70, .58, .50, .48, 1, .51,
    .53, .54, .47, .29, .50, .44, .42, .56, .38, .66, .59, .53, .51, 1),
  nrow = 14, ncol = 14)

R <- wais3[1:6,1:6]
b <- rellipsoid(R, Rsqr, Npoints = N)
b <- b$b
#
plot(b[1,], b[2,])
```

---

restScore

---

*Plot an ERF using rest scores*


---

**Description**

Plot an empirical response function using rest scores.

**Usage**

```
restScore(data, item, NCuts)
```

**Arguments**

data            N(subjects)-by-p(items) matrix of 0/1 item response data.  
 item            Generate a rest score plot for item item.  
 NCuts           Divide the rest scores into NCuts bins of equal width.

**Value**

A restscore plot with 95% confidence interval bars for the conditional probability estimates.

item            The item number.  
 bins            A vector of bin limits and bin sample sizes.  
 binProb        A vector of bin conditional probabilities.

**Author(s)**

Niels Waller

**Examples**

```
NSubj <- 2000

#generate sample k=1 FMP data
b <- matrix(c(
  #b0   b1   b2   b3   b4   b5 b6 b7 k
  1.675, 1.974, -0.068, 0.053, 0, 0, 0, 0, 1,
  1.550, 1.805, -0.230, 0.032, 0, 0, 0, 0, 1,
  1.282, 1.063, -0.103, 0.003, 0, 0, 0, 0, 1,
  0.704, 1.376, -0.107, 0.040, 0, 0, 0, 0, 1,
  1.417, 1.413, 0.021, 0.000, 0, 0, 0, 0, 1,
  -0.008, 1.349, -0.195, 0.144, 0, 0, 0, 0, 1,
  0.512, 1.538, -0.089, 0.082, 0, 0, 0, 0, 1,
  0.122, 0.601, -0.082, 0.119, 0, 0, 0, 0, 1,
  1.801, 1.211, 0.015, 0.000, 0, 0, 0, 0, 1,
  -0.207, 1.191, 0.066, 0.033, 0, 0, 0, 0, 1,
  -0.215, 1.291, -0.087, 0.029, 0, 0, 0, 0, 1,
  0.259, 0.875, 0.177, 0.072, 0, 0, 0, 0, 1,
  -0.423, 0.942, 0.064, 0.094, 0, 0, 0, 0, 1,
  0.113, 0.795, 0.124, 0.110, 0, 0, 0, 0, 1,
  1.030, 1.525, 0.200, 0.076, 0, 0, 0, 0, 1,
  0.140, 1.209, 0.082, 0.148, 0, 0, 0, 0, 1,
  0.429, 1.480, -0.008, 0.061, 0, 0, 0, 0, 1,
  0.089, 0.785, -0.065, 0.018, 0, 0, 0, 0, 1,
  -0.516, 1.013, 0.016, 0.023, 0, 0, 0, 0, 1,
  0.143, 1.315, -0.011, 0.136, 0, 0, 0, 0, 1,
  0.347, 0.733, -0.121, 0.041, 0, 0, 0, 0, 1,
  -0.074, 0.869, 0.013, 0.026, 0, 0, 0, 0, 1,
  0.630, 1.484, -0.001, 0.000, 0, 0, 0, 0, 1),
  nrow=23, ncol=9, byrow=TRUE)

data<-genFMPData(NSubj = NSubj, bParam = b, seed = 345)$data

## generate a rest score plot for item 12.
## the grey horizontal lines in the plot
## represent pseudo asymptotes that
## are significantly different from the
```

```
## (0,1) boundaries
restScore(data, item = 12, NCuts = 9)
```

rGivens

*Generate Correlation Matrices with Specified Eigenvalues*

## Description

rGivens generates correlation matrices with user-specified eigenvalues via a series of Givens rotations by methods described in Bendel & Mickey (1978) and Davis & Higham (2000).

## Usage

```
rGivens(eigs, Seed = NULL)
```

## Arguments

eigs	A vector of eigenvalues that must sum to the order of the desired correlation matrix. A fatal error will occur if <code>sum(eigs) != length(eigs)</code> .
Seed	Either a user supplied seed for the random number generator or 'NULL' for a function generated seed. Default Seed = 'NULL'.

## Value

R	A correlation matrix with desired spectrum.
Frob	The Frobenius norm of the difference between the initial and final matrices with the desired spectrum.
convergence	(Logical) TRUE if rGivens converged to a feasible solution, otherwise FALSE.

## References

Bendel, R. B. & Mickey, M. R. (1978). Population correlation matrices for sampling experiments, *Commun. Statist. Simulation Comput.*, B7, pp. 163-182.

Davies, P. I. & Higham, N. J. (2000). Numerically stable generation of correlation matrices and their factors, *BIT*, 40 (2000), pp. 640-651.

## Examples

```
## Example
## Generate a correlation matrix with user-specified eigenvalues

out <- rGivens(c(2.5, 1, 1, .3, .2), Seed = 123)

#> eigen(out$R)$values
#[1] 2.5 1.0 1.0 0.3 0.2

print(out)
#$R
#           [,1]      [,2]      [,3]      [,4]      [,5]
#[1,]  1.0000000 -0.1104098 -0.24512327  0.46497370  0.2392817
#[2,] -0.1104098  1.0000000  0.33564370 -0.46640155 -0.7645915
```

```

#[3,] -0.2451233  0.3356437  1.00000000 -0.02935466 -0.2024926
#[4,]  0.4649737 -0.4664016 -0.02935466  1.00000000  0.6225880
#[5,]  0.2392817 -0.7645915 -0.20249261  0.62258797  1.0000000
#
#Frob
#[1] 2.691613
#
##$0
#      [,1]      [,2]      [,3]      [,4]      [,5]
#[1,] 1.0349665  0.22537748 -0.46827121 -0.10448336 -0.24730565
#[2,] 0.2253775  0.31833805 -0.23208078  0.06591368 -0.14504161
#[3,] -0.4682712 -0.23208078  2.28911499  0.05430754  0.06964858
#[4,] -0.1044834  0.06591368  0.05430754  0.94884439 -0.14439623
#[5,] -0.2473056 -0.14504161  0.06964858 -0.14439623  0.40873606
#
#$convergence
#[1] TRUE

```

rMAP

*Generate Correlation Matrices with Specified Eigenvalues***Description**

rMAP uses the method of alternating projections (MAP) to generate correlation matrices with specified eigenvalues.

**Usage**

```
rMAP(eigenval, eps, maxits, Seed = NULL)
```

**Arguments**

eigenval	A vector of eigenvalues that must sum to the order of the desired correlation matrix. A fatal error will occur if <code>sum(eigenval) != length(eigenval)</code> .
eps	Convergence criterion. Default = 1e-12.
maxits	Maximm number of iterations of MAP.
Seed	Either a user supplied seed for the random number generator or 'NULL' for a function generated seed. Default Seed = 'NULL'.

**Value**

R	A correlation matrix with the desired spectrum.
evals	Eigenvalues of the returned matrix, R.
convergence	(Logical) TRUE if MAP converged to a feasible solution, otherwise FALSE.

**Author(s)**

Niels Waller

## References

Waller, N. G. (2016). Generating correlation matrices with specified eigenvalues using the method of alternating projections.

## Examples

```
## Example
## Generate a correlation matrix with user-specified eigenvalues

R <- rMAP(c(2.5, 1, 1, .3, .2), Seed = 123)$R
print(R, 2)

#      [,1] [,2] [,3] [,4] [,5]
#[1,] 1.000 0.5355 -0.746 -0.0688 -0.545
#[2,] 0.535 1.0000 -0.671 -0.0016 -0.056
#[3,] -0.746 -0.6711 1.000 0.0608 0.298
#[4,] -0.069 -0.0016 0.061 1.0000 0.002
#[5,] -0.545 -0.0564 0.298 0.0020 1.000

eigen(R)$values
#[1] 2.5 1.0 1.0 0.3 0.2
```

---

rmsd

*Root Mean Squared Deviation of (A - B)*


---

## Description

Calculates the root mean squared deviation of matrices A and B. If these matrices are symmetric (Symmetric = TRUE) then the calculation is based on the upper triangles of each matrix. When the matrices are symmetric, the diagonal of each matrix can be included or excluded from the calculation (IncludeDiag = FALSE)

## Usage

```
rmsd(A, B, Symmetric = TRUE, IncludeDiag = FALSE)
```

## Arguments

A	A possibly non square matrix.
B	A matrix of the same dimensions as matrix A.
Symmetric	Logical indicating whether A and B are symmetric matrices. (Default: Symmetric = TRUE)
IncludeDiag	Logical indicating whether to include the diagonals in the calculation. (Default: IncludeDiag = FALSE).

## Value

Returns the root mean squared deviation of (A - B).

**Author(s)**

Niels Waller

**Examples**

```
A <- matrix(rnorm(9), nrow = 3)
B <- matrix(rnorm(9), nrow = 3)

( rmsd(A, B, Symmetric = FALSE, IncludeDiag = TRUE) )
```

RnpdMAP

*Generate Random NPD R matrices from a user-supplied population R***Description**

Generate a list of Random NPD (pseudo) R matrices with a user-defined fixed minimum eigenvalue from a user-supplied population R using the method of alternating projections.

**Usage**

```
RnpdMAP(Rpop,
        Lp = NULL,
        NNegEigs = 1,
        NSmoothPosEigs = 4,
        NSubjects = NULL,
        NSamples = 0,
        MaxIts = 15000,
        PRINT=FALSE,
        Seed = NULL)
```

**Arguments**

Rpop	input (PD or PSD) p x p Population correlation matrix.
Lp	desired minimum eigenvalue in the NPD matrices.
NNegEigs	number of eigenvalues < 0 in Rnpd.
NSmoothPosEigs	number of eigenvalues > 0 to smooth: the smallest NSmoothPosEigs > 0 be smoothed toward 0.
NSubjects	sample size (required when NSamples > 0) parameter used to generate sample correlation matrices. Default = NULL.
NSamples	generate NSamples sample R matrices. If NSamples = 0 the program will attempt to find Rnpd such that   Rpop - Rnpd  _2 is minimized.
MaxIts	maximum number of projection iterations.
PRINT	(logical) If TRUE the program will print the iteration history for Lp. Default = NULL.
Seed	Optional seed for random number generation.

**Value**

Rpop	population (PD) correlation matrix.
R	sample correlation matrix.
Rnpd	NPD improper (pseudo) correlation matrix.
Lp	desired value of minimum eigenvalue.
minEig	observed value of minimum eigenvalue of Rnpd.
convergence	0 = converged; 1 = not converged in MaxIts iterations of the alternating projections algorithm.
feasible	logical) TRUE if $\max(\text{abs}(r_{ij})) \leq 1$ . If FALSE then one or more values in Rnpd > 1 in absolute value.
Seed	saved seed for random number generator.
prbs1	vector probabilities used to generate eigenvalues < 0.
prbs2	vector of probabilities used to smooth the smallest NSmoothPosEigs towards zero.

**Author(s)**

Niels G. Waller

**Examples**

```
library(MASS)

Nvar = 20
Nfac = 4
NSubj = 600
Seed = 123

set.seed(Seed)

## Generate a vector of classical item difficulties
p <- runif(Nvar)

cat("\nClassical Item Difficulties:\n")

print(rbind(1:Nvar,round(p,2)) )

summary(p)

## Convert item difficulties to quantiles
b <- qnorm(p)

## fnc to compute root mean squared standard deviation
RMSD <- function(A, B){
  sqrt(mean( ( A[lower.tri(A, diag = FALSE)] - B[lower.tri(B, diag = FALSE)] )^2))
}

## Generate vector of eigenvalues with clear factor structure
L <- eigGen(nDimensions = Nvar,
```



```

nMajorFactors = Nfac,
PrctMajor = .60,
threshold = .50)

## Generate a population R matrix with the eigenvalues in L
Rpop <- rGivens(eigs = L)$R

## Generate continuous data that will reproduce Rpop (exactly)
X <- mvrnorm(n = NSubj, mu = rep(0, Nvar),
             Sigma = Rpop, empirical = TRUE)

if( any(colSums(X) == 0) ){
  stop("One or more variables have zero variance. Generate a new data set.")
}

## Cut X at thresholds given in b to produce binary data U
U <- matrix(0, nrow(X), ncol(X))
for(j in 1:Nvar){
  U[X[,j] <= b[j],j] <- 1
}

## Compute tetrachoric correlations
Rtet <- tetcor(U, Smooth = FALSE, PRINT = TRUE)$r
# Calculate eigenvalues of tetrachoric R matrix
Ltet <- eigen(Rtet)$values

if(Ltet[Nvar] >= 0) stop("Rtet is P(S)D")

## Simulate NPD R matrix with minimum eigenvalue equal to
# min(Ltet)
out <- RnpdMAP(Rpop,
               Lp = Ltet[Nvar],
               NNegEigs = Nvar/5,
               NSmoothPosEigs = Nvar/5,
               NSubjects = 150,
               NSamples = 1,
               MaxIts = 15000,
               PRINT = FALSE,
               Seed = Seed)

## RLp is a NPD pseudo R matrix with min eigenvalue = min(Ltet)
RLp <- out[[1]]$Rnpd

## Calculate eigenvalues of simulated NPD R matrix (Rnpd)
Lnpd <- eigen(RLp, only.values = TRUE)$values

## Scree plots for observed and simulated NPD R matrices.
ytop <- max(c(L,Lnpd,Ltet))
pointSize = .8
plot(1:Nvar, L, typ = "b", col = "darkgrey", lwd=3,
     lty=1,
     main =
       "Eigenvalues of Rpop, Tet R, and Sim Tet R:
       \nSimulated vs Observed npd Tetrachoric R Matrices",
     ylim = c(-1, ytop),
     xlab = "Dimensions",

```

```

      ylab = "Eigenvalues",
      cex = pointSize, cex.main = 1.2)
points(1:Nvar, Lnpd, typ="b",
      col = "red", lwd = 3, lty=2, cex=pointSize)
points(1:Nvar, Ltet, typ="b",
      col = "darkgreen", lwd = 3, lty = 3, cex= pointSize)

legend("topright",
      legend = c("eigs Rpop", "eigs Sim Rnpd", "eigs Emp Rnpd"),
      col = c("darkgrey", "red", "darkgreen"),
      lty = c(1,2,3),
      lwd = c(4,4,4), cex = 1.5)

abline(h = 0, col = "grey", lty = 2, lwd = 4)

cat("\nRMSD(Rpop, Rtet) = ", round(rmsd(Rpop, Rtet), 3))
cat("\nRMSD(Rpop, RLp) = ", round(rmsd(Rpop, RLp), 3))

```

---

seBeta

*Standard Errors and CIs for Standardized Regression Coefficients*


---

## Description

Computes Normal Theory and ADF Standard Errors and CIs for Standardized Regression Coefficients

## Usage

```
seBeta(X, y, cov.x = NULL, cov.xy = NULL, var.y = NULL, Nobs = NULL,
alpha = 0.05, estimator = "ADF", digits = 3)
```

## Arguments

X	Matrix of predictor scores.
y	Vector of criterion scores.
cov.x	Covariance or correlation matrix of predictors.
cov.xy	Vector of covariances or correlations between predictors and criterion.
var.y	Criterion variance.
Nobs	Number of observations.
alpha	Desired Type I error rate; default = .05.
estimator	'ADF' or 'Normal' confidence intervals - requires raw X and raw y; default = 'ADF'.
digits	Number of significant digits to print; default = 3.

**Value**

cov.Beta	Normal theory or ADF covariance matrix of standardized regression coefficients.
se.Beta	standard errors for standardized regression coefficients.
alpha	desired Type-I error rate.
CI.Beta	Normal theory or ADF (1-alpha)% confidence intervals for standardized regression coefficients.
estimator	estimator = "ADF" or "Normal".

**Author(s)**

Jeff Jones and Niels Waller

**References**

Jones, J. A, and Waller, N. G. (2015). The Normal-Theory and Asymptotic Distribution-Free (ADF) covariance matrix of standardized regression coefficients: Theoretical extensions and finite sample behavior. *Psychometrika*, 80, 365-378.

**Examples**

```
library(MASS)

set.seed(123)

R <- matrix(.5, 3, 3)
diag(R) <- 1
X <- mvrnorm(n = 200, mu = rep(0, 3), Sigma = R, empirical = TRUE)
Beta <- c(.2, .3, .4)
y <- X%*% Beta + .64 * scale(rnorm(200))
seBeta(X, y, Nobs = 200, alpha = .05, estimator = 'ADF')

# 95% CIs for Standardized Regression Coefficients:
#
#      lbound estimate ubound
# beta_1  0.104      0.223  0.341
# beta_2  0.245      0.359  0.473
# beta_3  0.245      0.360  0.476
```

---

seBetaCor

*Standard Errors and CIs for Standardized Regression Coefficients  
from Correlations*

---

**Description**

Computes Normal Theory and ADF Standard Errors and CIs for Standardized Regression Coefficients from Correlations

**Usage**

```
seBetaCor(R, rxy, Nobs, alpha=.05, digits=3, covmat = 'normal')
```

**Arguments**

R	A $p \times p$ predictor correlation matrix.
rxxy	A $p \times 1$ vector of predictor-criterion correlations
Nobs	Number of observations.
alpha	Desired Type I error rate; default = .05.
digits	Number of significant digits to print; default = 3.
covmat	String = 'normal' (the default) or a $(p+1)p/2 \times (p+1)p/2$ covariance matrix of correlations. The default option computes an asymptotic covariance matrix under the assumption of multivariate normal data. Users can supply a covariance matrix under asymptotic distribution free (ADF) or elliptical distributions when available.

**Value**

cov.Beta	Covariance matrix of standardized regression coefficients.
se.Beta	Vector of standard errors for the standardized regression coefficients.
alpha	Type-I error rate.
CI.Beta	(1-alpha)% confidence intervals for standardized regression coefficients.

**Author(s)**

Jeff Jones and Niels Waller

**References**

- Jones, J. A, and Waller, N. G. (2013). The Normal-Theory and asymptotic distribution-free (ADF) covariance matrix of standardized regression coefficients: Theoretical extensions and finite sample behavior. Technical Report (052913)[TR052913]
- Nel, D.A.G. (1985). A matrix derivation of the asymptotic covariance matrix of sample correlation coefficients. *Linear Algebra and its Applications*, 67, 137-145.
- Yuan, K. and Chan, W. (2011). Biases and standard errors of standardized regression coefficients. *Psychometrika*, 76(4), 670–690.

**Examples**

```
R <- matrix(c(1.0000, 0.3511, 0.3661,
              0.3511, 1.0000, 0.4359,
              0.3661, 0.4359, 1.0000), 3, 3)

rxxy <- c(0.5820, 0.6997, 0.7621)
Nobs <- 46
out <- seBetaCor(R = R, rxxy = rxxy, Nobs = Nobs)

# 95% CIs for Standardized Regression Coefficients:
#
#      lbound estimate ubound
# beta_1 0.107      0.263 0.419
# beta_2 0.231      0.391 0.552
# beta_3 0.337      0.495 0.653
```

---

seBetaFixed	<i>Covariance Matrix and Standard Errors for Standardized Regression Coefficients for Fixed Predictors</i>
-------------	--

---

### Description

Computes Normal Theory Covariance Matrix and Standard Errors for Standardized Regression Coefficients for Fixed Predictors

### Usage

```
seBetaFixed(X, y, cov.x = NULL, cov.xy = NULL, var.y = NULL, var.error = NULL,
Nobs = NULL)
```

### Arguments

X	Matrix of predictor scores.
y	Vector of criterion scores.
cov.x	Covariance or correlation matrix of predictors.
cov.xy	Vector of covariances or correlations between predictors and criterion.
var.y	Criterion variance.
var.error	Optional argument to supply the error variance: var(y - yhat).
Nobs	Number of observations.

### Value

cov.Beta	Normal theory covariance matrix of standardized regression coefficients for fixed predictors.
se.Beta	Standard errors for standardized regression coefficients for fixed predictors.

### Author(s)

Jeff Jones and Niels Waller

### References

Yuan, K. & Chan, W. (2011). Biases and standard errors of standardized regression coefficients. *Psychometrika*, 76(4), 670-690.

### See Also

[seBeta](#)

## Examples

```
## We will generate some data and pretend that the Predictors are being held fixed

library(MASS)
R <- matrix(.5, 3, 3); diag(R) <- 1
Beta <- c(.2, .3, .4)

rm(list = ".Random.seed", envir = globalenv()); set.seed(123)
X <- mvrnorm(n = 200, mu = rep(0, 3), Sigma = R, empirical = TRUE)
y <- X %%% Beta + .64*scale(rnorm(200))

seBetaFixed(X, y)

# $covBeta
#           b1           b2           b3
# b1  0.003275127 -0.001235665 -0.001274303
# b2 -0.001235665  0.003037100 -0.001491736
# b3 -0.001274303 -0.001491736  0.002830157
#
# $seBeta
#           b1           b2           b3
# 0.05722872 0.05510989 0.05319922

## you can also supply covariances instead of raw data

seBetaFixed(cov.x = cov(X), cov.xy = cov(X, y), var.y = var(y), Nobs = 200)

# $covBeta
#           b1           b2           b3
# b1  0.003275127 -0.001235665 -0.001274303
# b2 -0.001235665  0.003037100 -0.001491736
# b3 -0.001274303 -0.001491736  0.002830157
#
# $seBeta
#           b1           b2           b3
# 0.05722872 0.05510989 0.05319922
```

---

skew

---

*Calculate Univariate Skewness for a Vector or Matrix*


---

## Description

Calculate univariate skewness for vector or matrix (algorithm G1 in Joanes & Gill, 1998).

## Usage

```
skew(x)
```

## Arguments

x Either a vector or matrix of numeric values.

Value

Skewness for each column in x.

Author(s)

Niels Waller

References

Joanes, D. N. & Gill, C. A. (1998). Comparing measures of sample skewness and kurtosis. *The Statistician*, 47, 183-189.

See Also

[kurt](#)

Examples

```
x <- matrix(rnorm(1000), 100, 10)
skew(x)
```

---

smoothAPA	<i>Smooth a NPD R matrix to PD using the Alternating Projection Algorithm</i>
-----------	---

---

Description

Smooth a Non positive definite (NPD) correlation matrix to PD using the Alternating Projection Algorithm with Dykstra’s correction via Theory described in Higham 2002.

Usage

```
smoothAPA(R, delta = 1e-06, fixR = NULL, Wghts = NULL, maxTries = 1000)
```

Arguments

R	A p x p indefinite matrix.
delta	Desired value of the smallest eigenvalue of smoothed matrix, RAPA. (Default = 1e-06).
fixR	User-supplied integer list that instructs the program to constrain elements in RAPA to equal corresponding elements in RAPA. For example if fixR = c(1,2) then smoothed matrix, RAPA[1:2,1:2] = R[1:2,1:2]. Default (fixR = NULL).
Wghts	A p-length vector of weights for differential variable weighting. Default (Wghts = NULL).
maxTries	Maximum number of iterations in the alternating projections algorithm. Default (maxTries = 1000).

**Value**

RAPA	A smoothed matrix.
delta	User-supplied delta value.
Wghts	User-supplied weight vector.
fixR	User-supplied integer list that instructs the program to constrain elements in RAPA to equal corresponding elements in R.
convergence	A value of 0 indicates that the algorithm located a feasible solution. A value of 1 indicates that no feasible solution was located within maxTries.

**Author(s)**

Niels Waller

**Examples**

```
data(BadRKtB)

#####
## Replicate analyses in Table 2 of Knol and ten Berge (1989).
#####

## n1 = 0,1
out<-smoothAPA(R = BadRKtB, delta = .0, fixR = NULL, Wghts = NULL, maxTries=1e06)
S <- out$RAPA
round(S - BadRKtB,3)
normF(S - BadRKtB)
eigen(S)$val

## n1 = 2
out<-smoothAPA(R = BadRKtB, fixR =c(1,2), delta=.0, Wghts = NULL, maxTries=1e06)
S <- out$RAPA
round(S - BadRKtB,3)
normF(S - BadRKtB)
eigen(S)$val

## n1 = 4
out<-smoothAPA(R = BadRKtB, fixR = 1:4, delta=.0, Wghts = NULL, maxTries=1e06)
S <- out$RAPA
round(S - BadRKtB,3)
normF(S - BadRKtB)
eigen(S)$val

## n1 = 5
out<-smoothAPA(R = BadRKtB, fixR = 1:5, delta=0, Wghts = NULL, maxTries=1e06)
S <- out$RAPA
round(S - BadRKtB,3)
normF(S - BadRKtB)
eigen(S)$val

#####
## Replicate analyses in Table 3 of Knol and ten Berge (1989).
#####

## n1 = 0,1
```



```

out<-smoothAPA(R = BadRKtB, delta = .05, fixR = NULL, Wghts = NULL, maxTries=1e06)
S <- out$RAPA
round(S - BadRKtB,3)
normF(S - BadRKtB)
eigen(S)$val

## n1 = 2
out<-smoothAPA(R = BadRKtB, fixR =c(1,2), delta=.05, Wghts = NULL, maxTries=1e06)
S <- out$RAPA
round(S - BadRKtB,3)
normF(S - BadRKtB)
eigen(S)$val

## n1 = 4
out<-smoothAPA(R = BadRKtB, fixR = 1:4, delta=.05, Wghts = NULL, maxTries=1e06)
S <- out$RAPA
round(S - BadRKtB,3)
normF(S - BadRKtB)
eigen(S)$val

## n1 = 5
out<-smoothAPA(R = BadRKtB, fixR = 1:5, delta=.05, Wghts = NULL, maxTries=1e06)
S <- out$RAPA
round(S - BadRKtB,3)
normF(S - BadRKtB)
eigen(S)$val

#####
## This example illustrates differential variable weighting.
##
## Imagine a scenerio in which variables 1 & 2 were collected with
## 5 times more subjects than variables 4 - 6 then . . .
#####
## n1 = 2
out<-smoothAPA(R = BadRKtB, delta=.0, fixR = NULL, Wghts = c(5, 5, rep(1,4)), maxTries=1e5)
S <- out$RAPA
round(S - BadRKtB,3)
normF(S - BadRKtB)
eigen(S)$val

```

---

smoothBY

---

*Smooth an NPD R matrix to PD using the Bentler Yuan 2011 method*


---

## Description

Smooth a NPD correlation matrix to PD using the Bentler and Yuan method.

## Usage

```
smoothBY(R, const = .98, eps = 1E-03)
```

**Arguments**

R	Indefinite Matrix.
const	const is a user-defined parameter that is defined as k in Bentler and Yuan (2011). If $0 < \text{const} < 1$ , then const is treated as a fixed value. If $\text{const} = 1$ then the program will attempt to find the highest value of const such that R is positive (semi) definite.
eps	If $\text{const} = 1$ then the program will iteratively reduce const by eps until either (a) the program converges or (b) $\text{const} \leq 0$ .

**Value**

RBY	smoothed correlation matrix.
constant	The final value of const.
convergence	(Logical) a value of TRUE indicates that the function converged.
outStatus	Convergence state for Rcsdp::csdp function.
	0:
	Success. Problem solved to full accuracy
	1:
	Success. Problem is primal infeasible
	2:
	Success. Problem is dual infeasible
	3:
	Partial Success. Solution found but full accuracy was not achieved
	4:
	Failure. Maximum number of iterations reached
	5:
	Failure. Stuck at edge of primal feasibility
	6:
	Failure. Stuck at edge of dual infeasibility
	7:
	Failure. Lack of progress
	8:
	Failure. X or Z (or Newton system O) is singular

9:

Failure. Detected NaN or Inf values

glb            Greatest lower bound reliability estimates.  
 eps           Default value (eps = 1E-03) or user-supplied value of eps.

### Author(s)

Code modified from that reported in Debelak, R. & Tran, U. S. (2011).

### References

Bentler, P. M. & Yuan, K. H. (2011). Positive definiteness via off-diagonal scaling of a symmetric indefinite matrix. *Psychometrika*, 76(1), 119–123.

Debelak, R. & Tran, U. S. (2013). Principal component analysis of smoothed tetrachoric correlation matrices as a measure of dimensionality. *Educational and Psychological Measurement*, 73(1), 63–77.

### Examples

```
data(BadRBY)

out<-smoothBY(R = BadRBY, const = .98)
cat("\nSmoothed Correlation Matrix\n")
print( round(out$RBY,8) )
cat("\nEigenvalues of smoothed matrix\n")
print( eigen(out$RBY)$val )
```

---

smoothKB

*Smooth a Non PD Correlation Matrix*

---

### Description

A function for smoothing a non-positive definite correlation matrix by the method of Knol and Berger (1991).

### Usage

```
smoothKB(R, eps = 1E8 * .Machine$double.eps)
```

### Arguments

R            A non-positive definite correlation matrix.  
 eps           Small positive number to control the size of the non-scaled smallest eigenvalue of the smoothed R matrix. Default = 1E8 \* .Machine\$double.eps

### Value

RKB           A Smoothed (positive definite) correlation matrix.  
 eps           Small positive number to control the size of the non-scaled smallest eigenvalue of the smoothed R matrix.

**Author(s)**

Niels Waller

**References**

Knol, D. L., & Berger, M. P. F., (1991). Empirical comparison between factor analysis and multidimensional item response models. *Multivariate Behavioral Research*, 26, 457-477.

**Examples**

```
data(BadRLG)

## RKB = smoothed R
RKB<-smoothKB(R=BadRLG, eps = 1E8 * .Machine$double.eps)$RKB
print(eigen(RKB)$values)
```

smoothLG

*Smooth NPD to Nearest PSD or PD Matrix***Description**

Smoothing an indefinite matrix to a PSD matrix via theory described by Lurie and Goldberg

**Usage**

```
smoothLG(R, start.val = NULL, Wghts = NULL, PD = FALSE,
Penalty = 50000, eps=1e-07)
```

**Arguments**

R	Indefinite Matrix.
start.val	Optional vector of start values for Cholesky factor of S.
Wghts	An optional matrix of weights such that the objective function minimizes $w_{ij}(r_{ij} - s_{ij})^2$ , where $w_{ij}$ is $Wghts[i,j]$ .
PD	Logical (default = FALSE). If PD = TRUE then the objective function will smooth the least squares solution to insure Positive Definiteness.
Penalty	A scalar weight to scale the Lagrangian multiplier. Default = 50000.
eps	A small value to add to zero eigenvalues if smoothed matrix must be PD. Default = 1e-07.

**Value**

RLG	Lurie Goldberg smoothed matrix.
RKB	Knol and Berger smoothed matrix.
convergence	0 = converged solution, 1 = convergence failure.
start.val	Vector of start.values.
gr	Analytic gradient at solution.

Penalty	Scalar used to scale the Lagrange multiplier.
PD	User-supplied value of PD.
Wghts	Weights used to scale the squared euclidean distances.
eps	Value added to zero eigenvalue to produce PD matrix.

**Author(s)**

Niels Waller

**Examples**

```
data(BadRLG)

out<-smoothLG(R = BadRLG, Penalty = 50000)
cat("\nGradient at solution:", out$gr,"\n")
cat("\nNearest Correlation Matrix\n")
print( round(out$RLG,8) )

#####
## Rousseeuw Molenbergh example
data(BadRRM)

out <- smoothLG(R = BadRRM, PD=TRUE)
cat("\nGradient at solution:", out$gr,"\n")
cat("\nNearest Correlation Matrix\n")
print( round(out$RLG,8) )

## Weights for the weighted solution
W <- matrix(c(1, 1, .5,
              1, 1, 1,
              .5, 1, 1), nrow = 3, ncol = 3)
tmp <- smoothLG(R = BadRRM, PD = TRUE, eps=.001)
cat("\nGradient at solution:", out$gr,"\n")
cat("\nNearest Correlation Matrix\n")
print( round(out$RLG,8) )
print( eigen(out$RLG)$val )

## Rousseeuw Molenbergh
## non symmetric matrix
T <- matrix(c(.8, -.9, -.9,
              -1.2, 1.1, .3,
              -.8, .4, .9), nrow = 3, ncol = 3,byrow=TRUE)
out <- smoothLG(R = T, PD = FALSE, eps=.001)

cat("\nGradient at solution:", out$gr,"\n")
cat("\nNearest Correlation Matrix\n")
print( round(out$RLG,8) )
```

**Description**

summary method for class "monte"

**Usage**

```
## S3 method for class 'monte'
summary(object, digits = 3, compute.validities = FALSE, Total.stats=TRUE, ...)
```

**Arguments**

<code>object</code>	An object of class <code>monte</code> , usually, a result of a call to <code>monte</code> .
<code>digits</code>	Number of digits to print. Default = 3.
<code>compute.validities</code>	Logical: If TRUE then the program will calculate the indicator validities ( $\eta^2$ ) for the generated data.
<code>Total.stats</code>	Logical: If TRUE then the program will return the following statistics for the total sample: (1) indicator correlation matrix, (2) indicator skewness, (3) indicator kurtosis.
<code>...</code>	Optional arguments.

**Value**

Various descriptive statistics will be computed within groups including"

1. `clus.size` Number of objects within each group.
2. `centroids` Group centroids.
3. `var.matrix` Within group variances.
4. Ratio of within group variances (currently printed but not saved).
5. `cor.list` Expected within group correlations.
6. `obs.cor` Observed within group correlations.
7. `skew.list` Expected within group indicator skewness values.
8. `obs.skew` Observed within group indicator skewness values.
9. `kurt.list` Expected within group indicator kurtosis values.
10. `obs.kurt` Observed within group indicator kurtosis values.
11. `validities` Observed indicator validities.
12. `Total.cor` Total sample correlation matrix.
13. `Total.skew` Total sample indicator skewness.
14. `Total.kurt` Total sample indicator kurtosis.

**Examples**

```
## set up a 'monte' run for the Fisher iris data

sk.lst <- list(c(0.120, 0.041, 0.106, 1.254),
               c(0.105, -0.363, -0.607, -0.031),
               c(0.118, 0.366, 0.549, -0.129) )

kt.lst <- list(c(-0.253, 0.955, 1.022, 1.719),
               c(-0.533, -0.366, 0.048, -0.410),
               c(0.033, 0.706, -0.154, -0.602))

cormat <- lapply(split(iris[,1:4], iris[,5]), cor)
```

```
my.iris <- monte(seed = 123, nvar = 4, nclus = 3, cor.list = cormat,
                 clus.size = c(50, 50, 50),
                 eta2 = c(0.619, 0.401, 0.941, 0.929),
                 random.cor = FALSE,
                 skew.list = sk.lst, kurt.list = kt.lst,
                 secor = .3,
                 compactness = c(1, 1, 1),
                 sortMeans = TRUE)
summary(my.iris)
```

summary.monte1

*Summary Method for an Object of Class Monte1***Description**

summary method for class "monte1"

**Usage**

```
## S3 method for class 'monte1'
summary(object, digits=3, ...)
```

**Arguments**

object	An object of class monte1, usually, a result of a call to monte1.
digits	Number of significant digits to print in final results.
...	Additional argument affecting the summary produced.

**Value**

Various descriptive statistics will be computed including"

1. Expected correlation matrix.
2. Observed correlation matrix.
3. Expected indicator skewness values.
4. Observed indicator skewness values.
5. Expected indicator kurtosis values.
6. Observed indicator kurtosis values.

**Examples**

```
## Generate dimensional data for 4 variables.
## All correlations = .60; all variable
## skewness = 1.75;
## all variable kurtosis = 3.75

cormat <- matrix(.60, 4, 4)
diag(cormat) <- 1

nontaxon.dat <- monte1(seed = 123, nsub = 100000, nvar = 4, skewvec = rep(1.75, 4),
                      kurtvec = rep(3.75, 4), cormat = cormat)

summary(nontaxon.dat)
```

svdNorm

*Compute theta surrogates via normalized SVD scores***Description**

Compute theta surrogates by calculating the normalized left singular vector of a (mean-centered) data matrix.

**Usage**

```
svdNorm(data)
```

**Arguments**

data                    N(subjects)-by-p(items) matrix of 0/1 item response data.

**Value**

the normalized left singular vector of the mean centered data matrix.  
svdNorm will center the data automatically.

**Author(s)**

Niels Waller

**Examples**

```
NSubj <- 2000

## example item parameters for sample data: k=1 FMP
b <- matrix(c(
  #b0   b1    b2    b3      b4    b5 b6 b7  k
  1.675, 1.974, -0.068, 0.053, 0, 0, 0, 0, 1,
  1.550, 1.805, -0.230, 0.032, 0, 0, 0, 0, 1,
  1.282, 1.063, -0.103, 0.003, 0, 0, 0, 0, 1,
  0.704, 1.376, -0.107, 0.040, 0, 0, 0, 0, 1,
  1.417, 1.413, 0.021, 0.000, 0, 0, 0, 0, 1,
  -0.008, 1.349, -0.195, 0.144, 0, 0, 0, 0, 1,
  0.512, 1.538, -0.089, 0.082, 0, 0, 0, 0, 1,
  0.122, 0.601, -0.082, 0.119, 0, 0, 0, 0, 1,
  1.801, 1.211, 0.015, 0.000, 0, 0, 0, 0, 1,
  -0.207, 1.191, 0.066, 0.033, 0, 0, 0, 0, 1,
  -0.215, 1.291, -0.087, 0.029, 0, 0, 0, 0, 1,
  0.259, 0.875, 0.177, 0.072, 0, 0, 0, 0, 1,
  -0.423, 0.942, 0.064, 0.094, 0, 0, 0, 0, 1,
  0.113, 0.795, 0.124, 0.110, 0, 0, 0, 0, 1,
  1.030, 1.525, 0.200, 0.076, 0, 0, 0, 0, 1,
  0.140, 1.209, 0.082, 0.148, 0, 0, 0, 0, 1,
  0.429, 1.480, -0.008, 0.061, 0, 0, 0, 0, 1,
  0.089, 0.785, -0.065, 0.018, 0, 0, 0, 0, 1,
  -0.516, 1.013, 0.016, 0.023, 0, 0, 0, 0, 1,
  0.143, 1.315, -0.011, 0.136, 0, 0, 0, 0, 1,
  0.347, 0.733, -0.121, 0.041, 0, 0, 0, 0, 1,
  -0.074, 0.869, 0.013, 0.026, 0, 0, 0, 0, 1,
```



```

0.630, 1.484, -0.001, 0.000, 0, 0, 0, 0, 1),
nrow=23, ncol=9, byrow=TRUE)

# generate data using the above item paramters
data<-genFMPData(NSubj=NSubj, bParam=b, seed=345)$data

# compute (initial) surrogate theta values from
# the normed left singular vector of the centered
# data matrix
thetaInit<-svdNorm(data)

```

tetcor

*Compute ML Tetrachoric Correlations***Description**

Compute ML tetrachoric correlations with optional bias correction and smoothing.

**Usage**

```
tetcor(X, y = NULL, BiasCorrect, stderr, Smooth = TRUE, max.iter, PRINT = TRUE)
```

**Arguments**

X	Either a matrix or vector of (0/1) binary data.
y	An optional(if X is a matrix) vector of (0/1) binary data.
BiasCorrect	A logical that determines whether bias correction (Brown & Benedetti, 1977) is performed. Default = TRUE.
stderr	A logical that determines whether standard errors are calculated. Default = FALSE.
Smooth	A logical which determines whether the tetrachoric correlation matrix should be smoothed. A smoothed matrix is always positive definite.
max.iter	Maximum number of iterations. Default = 50.
PRINT	A logical that determines whether to print progress updates during calculations. Default = TRUE

**Value**

If stderr = FALSE, tetcor returns a matrix of tetrachoric correlations. If stderr = TRUE then tetcor returns a list the first component of which is a matrix of tetrachoric correlations and the second component is a matrix of standard errors (see Hamdan, 1970).

**Author(s)**

Niels Waller

**References**

Brown, M. B. & Benedetti, J. K. (1977). On the mean and variance of the tetrachoric correlation coefficient. *Psychometrika*, 42, 347–355.

Divgi, D. R. (1979) Calculation of the tetrachoric correlation coefficient. *Psychometrika*, 44, 169-172.

Hamdan, M. A. (1970). The equivalence of tetrachoric and maximum likelihood estimates of rho in 2 by 2 tables. *Biometrika*, 57, 212-215.

## Examples

```
## generate bivariate normal data
library(MASS)
set.seed(123)
rho <- .85
xy <- mvrnorm(100000, mu = c(0,0), Sigma = matrix(c(1, rho, rho, 1), ncol = 2))

# dichotomize at difficulty values
p1 <- .7
p2 <- .1
xy[,1] <- xy[,1] < qnorm(p1)
xy[,2] <- xy[,2] < qnorm(p2)

print( apply(xy,2,mean), digits = 2)
#[1] 0.700 0.099

tetcor(X = xy, BiasCorrect = TRUE,
       stderr = TRUE, Smooth = TRUE, max.iter = 5000)

# $r
# [,1]      [,2]
# [1,] 1.0000000 0.8552535
# [2,] 0.8552535 1.0000000
#
# $se
# [,1]      [,2]
# [1,] NA      0.01458171
# [2,] 0.01458171 NA
#
# $Warnings
# list()
```

---

tetcorQuasi

*Correlation between a Naturally and an Artificially Dichotomized Variable*

---

## Description

A function to compute Ulrich and Wirtz's correlation of a naturally and an artificially dichotomized variable.

## Usage

```
tetcorQuasi(x, y = NULL)
```

## Arguments

**x** An N x 2 matrix or an N x 1 vector of binary responses coded 0/1.

**y** An optional (if x is a vector) vector of 0/1 responses.

**Value**

A quasi tetrachoric correlation  
...

**Author(s)**

Niels Waller

**References**

Ulrich, R. & Wirtz, M. (2004). On the correlation of a naturally and an artificially dichotomized variable. *British Journal of Mathematical and Statistical Psychology*, 57, 235-252.

**Examples**

```
set.seed(321)
Nsubj <- 5000

## Generate mvn data with rxy = .5
R <- matrix(c(1, .5, .5, 1), 2, 2)
X <- MASS::mvrnorm(n = Nsubj, mu = c(0, 0), Sigma = R, empirical = TRUE)

## dichotomize data
thresholds <- qnorm(c(.2, .3))
binaryData <- matrix(0, Nsubj, 2)

for(i in 1:2){
  binaryData[X[,i] <= thresholds[i],i] <- 1
}

## calculate Pearson correlation
cat("\nPearson r: ", round(cor(X)[1,2], 2))

## calculate Pearson Phi correlation
cat("\nPhi r: ", round(cor(binaryData)[1,2], 2))

## calculate tetrachoric correlation
cat("\nTetrachoric r: ", round(tetcor(binaryData)$r[1,2], 2))

## calculate Quasi-tetrachoric correlation
cat("\nQuasi-tetrachoric r: ", round(tetcorQuasi(binaryData), 2))
```

---

vcos

---

*Compute the Cosine Between Two Vectors*


---

**Description**

Compute the cosine between two vectors.

**Usage**

```
vcos(x,y)
```

**Arguments**

x	A p x 1 vector.
y	A p x 1 vector.

**Value**

Cosine between x and y

**Examples**

```
x <- rnorm(5)
y <- rnorm(5)
vcos(x, y)
```

---

vnorm	<i>Norm a Vector to Unit Length</i>
-------	-------------------------------------

---

**Description**

Norm a vector to unit length.

**Usage**

```
vnorm(x)
```

**Arguments**

x	An n by 1 vector.
---	-------------------

**Value**

the scaled (i.e., unit length) input vector

**Author(s)**

Niels Waller

**Examples**

```
x <- rnorm(5)
v <- vnorm(x)
print(v)
```

# Index

## \*Topic **BiFAD**

BiFAD, [8](#)

## \*Topic **Statistics**

adfCor, [3](#)

adfCov, [4](#)

corSmooth, [13](#)

d2r, [14](#)

eigGen, [16](#)

faAlign, [19](#)

fals, [22](#)

kurt, [50](#)

normalCor, [59](#)

r2d, [61](#)

rcor, [64](#)

seBeta, [74](#)

seBetaCor, [75](#)

seBetaFixed, [77](#)

skew, [78](#)

smoothKB, [83](#)

tetcor, [89](#)

tetcorQuasi, [90](#)

vcos, [91](#)

## \*Topic **Statistics**

faSort, [25](#)

## \*Topic **datagen**

bigen, [10](#)

corSample, [12](#)

enhancement, [17](#)

genCorr, [43](#)

monte, [51](#)

monte1, [58](#)

rarc, [62](#)

rcone, [63](#)

rellipsoid, [65](#)

rGivens, [68](#)

rMAP, [69](#)

## \*Topic **datasets**

HS9Var, [46](#)

## \*Topic **fungible**

faAlign, [19](#)

fungible, [30](#)

fungibleExtrema, [31](#)

fungibleL, [33](#)

fungibleR, [35](#)

RnpdMAP, [71](#)

## \*Topic **statistics**

BadRBY, [5](#)

BadRJN, [6](#)

BadRKtB, [6](#)

BadRLG, [7](#)

BadRRM, [7](#)

eap, [14](#)

erf, [18](#)

faMAP, [23](#)

FMP, [26](#)

FMPMonotonicityCheck, [29](#)

FUP, [39](#)

gen4PMDData, [42](#)

genFMPData, [44](#)

irf, [47](#)

itemDescriptives, [48](#)

normF, [60](#)

restScore, [66](#)

smoothAPA, [79](#)

smoothBY, [81](#)

smoothLG, [84](#)

svdNorm, [88](#)

vnorm, [92](#)

## \*Topic **stats**

rmsd, [70](#)

adfCor, [3](#), [60](#)

adfCov, [4](#)

BadRBY, [5](#)

BadRJN, [6](#)

BadRKtB, [6](#)

BadRLG, [7](#)

BadRRM, [7](#)

BiFAD, [8](#)

bigen, [10](#)

corSample, [12](#)

corSmooth, [13](#)

d2r, [14](#)

eap, [14](#)

eigGen, 16  
enhancement, 17  
erf, 18  
  
faAlign, 19  
fals, 22, 25  
faMAP, 23  
faSort, 25  
FMP, 26  
FMPMonotonicityCheck, 29  
fungible, 30  
fungibleExtrema, 31  
fungibleL, 33  
fungibleR, 35  
FUP, 39  
  
gen4PMDData, 42  
genCorr, 43, 65  
genFMPData, 44  
  
HS9Var, 46  
  
irf, 47  
itemDescriptives, 48  
  
kurt, 50, 79  
  
monte, 51, 59  
monte1, 58  
  
normalCor, 59  
normF, 60  
  
plot.monte, 61  
print.summary.monte(summary.monte), 85  
print.summary.monte1(summary.monte1),  
87  
  
r2d, 61  
rarc, 62  
rcone, 63  
rcor, 64  
rellipsoid, 65  
restScore, 66  
rGivens, 68  
rMAP, 69  
rmsd, 70  
RnpdMAP, 71  
  
seBeta, 74, 77  
seBetaCor, 75  
seBetaFixed, 77  
skew, 51, 78  
smoothAPA, 79  
smoothBY, 81  
smoothKB, 83  
smoothLG, 84  
summary.monte, 59, 85  
summary.monte1, 59, 87  
svdNorm, 26, 40, 88  
  
tetcor, 89  
tetcorQuasi, 90  
  
vcos, 91  
vnorm, 92