

# Finding the Lovasz Number

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The Lovasz Number of a graph  $\mathbf{G}$ , denoted  $\vartheta(\mathbf{G})$ , is the upper bound on the Shannon capacity of the graph ([1]). For an adjacency matrix  $\mathbf{B} = [B_{ij}]$  the problem of finding the Lovasz number is given by the following primal SQLP problem

$$\begin{aligned} & \underset{\mathbf{X}}{\text{minimize}} && \text{tr}(\mathbf{C}\mathbf{X}) \\ & \text{subject to} && \begin{aligned} \text{tr}(\mathbf{X}) &= 1 \\ X_{ij} &= 0 \quad \text{if } B_{ij} = 1 \\ \mathbf{X} &\in \mathcal{S}^n \end{aligned} \end{aligned}$$

The function `lovasz` takes as input an adjacency matrix  $\mathbf{B}$ , and returns the input variables necessary for the Lovasz number to be found using `sqlp`.

```
R> out <- lovasz(B)
R> blk <- out$blk
R> At <- out$At
R> C <- out$C
R> b <- out$b

R> sqlp(blk,At,C,b)
```

## Numerical Example

To compute the Lovasz number using `sqlp`, we need only the (weighted) adjacency matrix representing a graph object.

```
R> data(Glovasz)
```

	V1	V2	V3	V4	V5	V6	V7	V8	V9	V10
[1,]	0	0	0	1	0	0	1	1	0	0
[2,]	0	0	0	1	0	0	1	0	1	1
[3,]	0	0	0	0	0	0	0	1	0	0
[4,]	1	1	0	0	0	0	0	1	0	1
[5,]	0	0	0	0	0	0	1	1	1	1
[6,]	0	0	0	0	0	0	0	0	1	0
[7,]	1	1	0	0	1	0	0	1	1	1
[8,]	1	0	1	1	1	0	1	0	0	0
[9,]	0	1	0	0	1	1	1	0	0	1
[10,]	0	1	0	1	1	0	1	0	1	0

The Lovasz number for the associated graph is the value of the primal objective function. Again, since the objective function was negated to make the primal problem a minimization, we negate the value of the objective function.

```
R> out <- lovasz(Glovasz)
R> blk <- out$blk
R> At <- out$At
R> C <- out$C
R> b <- out$b

R> out <- sqlp(blk,At,C,b)
R> -out$pobj

[1] 5
```

## References

- [1] László Lovász. On the shannon capacity of a graph. *IEEE Transactions on Information theory*, 25(1):1–7, 1979.