0.1 `ei.RxC`: Hierarchical Multinomial-Dirichlet Ecological Inference Model for $R \times C$ Tables

Given n contingency tables, each with observed marginals (column and row totals), ecological inference (EI) estimates the internal cell values in each table. The hierarchical Multinomial-Dirichlet model estimates cell counts in $R \times C$ tables. The model is implemented using a nonlinear least squares approximation and, with bootstrapping for standard errors, had good frequentist properties.

Syntax

```
> z.out <- zelig(cbind(T0, T1, T2, T3) ~ X0 + X1,
                 covar = NULL,
                 model = "eiRxC", data = mydata)
> x.out <- setx(z.out, fn = NULL)
> s.out <- sim(z.out)
```

Inputs

- `T0`, `T1`, `T2`,..., `TC`: numeric vectors (either counts, or proportions that sum to one for each row) containing the column margins of the units to be analyzed.
- `X0`, `X1`, `X2`,...,`XR`: numeric vectors (either counts, or proportions that sum to one for each row) containing the row margins of the units to be analyzed.
- `covar`: (optional) a covariate that varies across tables, specified as `covar = ~ Z1`, for example. (The model only accepts one covariate.)

Examples

1. Basic examples: No covariate
   Attaching the example dataset:

   ```
   > data(Weimar)
   ```

   Estimating the model:

   ```
   > z.out <- zelig(cbind(Nazi, Government, Communists, FarRight,
   +     Other) ~ shareunemployed + shareblue + sharewhite + shareself +
   +     sharedomestic, model = "ei.RxC", data = Weimar)
   > summary(z.out)
   ```

   Setting values for in-sample simulations given marginal values:

   ```
   > x.out <- setx(z.out)
   ```

Estimate fractions of different social groups that support political parties:

```
> s.out <- sim(z.out)
```

Summarizing fractions of different social groups that support political parties:

```
> summary(s.out)
```

2. Example of covariates being present in the model

   Using the example dataset Weimar and estimating the model

```
> z.out <- zelig(cbind(Nazi, Government, Communists, FarRight,
+     Other) ~ shareunemployed + shareblue + sharewhite + shareself +
+     sharedomestic, covar = ~shareprotestants, model = "ei.RxC",
+     data = Weimar)
> summary(z.out)
```

   Set the covariate to its default (mean/median) value

```
> x.out <- setx(z.out)
```

   Estimate fractions of different social groups that support political parties:

```
> s.out <- sim(z.out)
```

   Summarizing fractions of different social groups that support political parties:

```
> summary(s.out)
```

Model

Consider the following 5×5 contingency table for the voting patterns in Weimar Germany. For each geographical unit i ($i = 1, \ldots, p$), the marginals $T_{1i}, \ldots, T_{Ci}$, $X_{1i}, \ldots, X_{Ri}$ are known for each of the p electoral precincts, and we would like to estimate ($\beta_i^{rc}, r = 1, \ldots, R, c = 1, \ldots, C-1$) which are the fractions of people in social class r who vote for party c, for all r and c.

	Nazi	Government	Communists	Far Right	Other	
Unemployed	β^i_{11}	β^i_{12}	β^i_{13}	β^i_{14}	$1 - \sum_{c=1}^4 \beta^i_{1c}$	X^i_1
Blue	β^i_{21}	β^i_{22}	β^i_{23}	β^i_{24}	$1 - \sum_{c=1}^4 \beta^i_{2c}$	X^i_2
White	β^i_{31}	β^i_{32}	β^i_{33}	β^i_{34}	$1 - \sum_{c=1}^4 \beta^i_{3c}$	X^i_3
Self	β^i_{41}	β^i_{42}	β^i_{43}	β^i_{44}	$1 - \sum_{c=1}^4 \beta^i_{4c}$	X^i_4
Domestic	β^i_{51}	β^i_{52}	β^i_{53}	β^i_{54}	$1 - \sum_{c=1}^4 \beta^i_{5c}$	X^i_5
	T_{1i}	T_{2i}	T_{3i}	T_{4i}	$1 - \sum_{c=1}^4 \beta_{ci}$	

The marginal values $X_{1i}, \ldots, X_{Ri}$, $T_{1i}, \ldots, T_{Ci}$ may be observed as counts or fractions.

Let $T_i^{'} = (T_{1i}^{'}, T_{2i}^{'}, \ldots, T_{Ci}^{'})$ be the number of voting age persons who turn out to vote for different parties. There are three levels of hierarchy in the Multinomial-Dirichlet EI model. At the first stage, we model the data as:

- The *stochastic component* is described $T_i^{'}$ which follows a multinomial distribution:

$$T_i^{'} \sim \text{Multinomial}(\Theta_{1i}, \ldots, \Theta_{Ci})$$

- The *systematic components* are

$$\Theta_{ci} = \sum_{r=1}^{R} \beta^i_{rc} X_{ri} \quad \text{for} \quad c = 1, \ldots, C$$

At the second stage, we use an optional covariate to model Θ_{ci}'s and β^i_{qrc}:

- The *stochastic component* is described by $\beta^i_r = (\beta_{r1}, \beta_{r2}, \ldots, \beta_{r,C-1})$ for $i = 1, \ldots, , p$ and $r = 1, \ldots, R$, which follows a Dirichlet distribution:

$$\beta^i_r \sim \text{Dirichlet}(\alpha^i_{r1}, \ldots, \alpha^i_{rc})$$

- The *systematic components* are

$$\alpha^i_{rc} = \frac{d_r exp(\gamma_{rc} + \delta_{rc} Z_i)}{d_r(1 + \sum_{j=1}^{C-1} exp(\gamma_{rj} + \delta_{rj} Z_i))} = \frac{exp(\gamma_{rc} + \delta_{rc} Z_i)}{1 + \sum_{j=1}^{C-1} exp(\gamma_{rj} + \delta_{rj} Z_i)}$$

  for $i = 1, \ldots, p$, $r = 1, \ldots, R$, and $c = 1, \ldots, C-1$.

  In the third stage, we assume that the regression parameters (the γ_{rc}'s and δ_{rc}'s) are *a priori* independent, and put a flat prior on these regression parameters. The parameters d_r for $r = 1, \ldots, R$ are assumed to follow exponential distributions with mean $\frac{1}{\lambda}$.

Output Values

The output of each Zelig command contains useful information which you may view. For example, if you run

```
> z.out <- zelig(cbind(T0, T1, T2) ~ X0 + X1 + X2,
          model = "eiRxC", data = mydata)
```

then you may examine the available information in `z.out` by using `names(z.out)`. For example,

- From the `zelig()` output object `z.out$coefficients` are the estimates of γ_{ij} (and also δ_{ij}, if covariates are present). The parameters are returned as a single vector of length $R \times (C-1)$. If there is a covariate, δ is concatenated to it.
- From the `sim()` output object, you may extract the parameters β_{ij} corresponding to the estimated fractions of different social groups that support different political parties, by using `s.out$qi$ev`. For each precinct, that will be a matrix with dimensions: simulations $\times R \times C$.

How to Cite

To cite the *ei.RxC* Zelig model use:

> Jason Wittenberg, Ferdinand Alimadhi, and Olivia Lau. 2007. "ei.RxC:Hierarchical Multinomial-Dirichlet Ecological Inference Model," in Kosuke Imai,Gary King, and Olivia Lau, "Zelig: Everyone's Statistical Software," `http://gking.harvard.edu/zelig`.

To cite Zelig as a whole, please reference these two sources:

> Kosuke Imai, Gary King, and Olivia Lau. 2007. "Zelig: Everyone's Statistical Software," `http://GKing.harvard.edu/zelig`.

> Kosuke Imai, Gary King, and Olivia Lau. 2007. "Toward A Common Framework for Statistical Analysis and Development," `http://gking.harvard.edu/files/abs/z-abs.shtml`.

See also

For more information please see Rosen et al. (2001)

Bibliography

Rosen, O., Jiang, W., King, G., and Tanner, M. A. (2001), "Bayesian and Frequentist Inference for Ecological Inference: The $R \times C$ Case," *Statistica Neerlandica*, 55, 134–156, http://gking.harvard.edu/files/abs/rosen-abs.shtml.