

Highfrequency: Toolkit for the analysis of highfrequency financial data in R.

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Abstract

The highfrequency package contains an extensive toolkit for the use of highfrequency financial data in R. It offers functionality to manage, clean and match highfrequency trades and quotes data. Furthermore, it enables users to: calculate easily various liquidity measures, estimate and forecast volatility, and investigate microstructure noise and intraday periodicity. More information on: <http://highfrequency.herokuapp.com>.

Coming soon: an interactive tutorial on <http://www.datamind.org>.

Keywords: High-frequency data, Liquidity, Quotes, R software, Realized volatility, Trades.

1. General information

The economic value of analyzing *high-frequency financial* data is now obvious, both in the academic and financial world. It is the basis of intraday and daily risk monitoring and forecasting, an input to the portfolio allocation process, and also for high-frequency trading. For the efficient use of high-frequency data in financial decision making, the **highfrequency** package implements state of the art techniques for cleaning and matching for trades and quotes, as well as the calculation and forecasting of liquidity, realized and correlation measures based on high-frequency data. The **highfrequency** package is the outcome of a Google summer of code project and an improved and updated version of the 2 packages: **RTAQ** (Cornelissen and Boudt 2012) and **realized** (Payseur 2008).

Handling high-frequency data can be particularly challenging because of the specific characteristics of the data, as extensively documented in Yan and Zivot (2003). The **highfrequency** package offers high-level tools for the analysis of high-frequency data. These tools tackle three typical challenges of working with high-frequency data. A first specific challenge is the enormous number of observations, that can reach heights of millions observations per stock per day. Secondly, transaction-by-transaction data is by nature irregularly spaced over time. Thirdly, the recorded data often contains errors for various reasons.

The **highfrequency** package offers a (basic) interface to manage highfrequency trades and quotes data. Furthermore and foremost, it offers easy-to-use tools to clean and aggregate high-frequency data, to calculate liquidity measures and, measure and forecast volatility. The tools

in `highfrequency` are designed in such a way that they will work with most data input types. The package contains an extensive collection of functions to calculate realized measures, since it arose by merging the package `RTAQ` (Cornelissen and Boudt 2012) from the TradeAnalytics project and the package `realized` (Payseur 2008).

`highfrequency` strongly builds on the functionality offered by the `xts` package (Ryan and Ulrich 2009) package. You can use the functionality in the `highfrequency` package for trades and quotes objects from the most popular vendors, as long as your objects are stored as `xts` objects.

The source code of the `highfrequency` package is hosted on github and the latest version of the package can be downloaded through the following command:

```
library("devtools"); install_github("highfrequency", "jonathancornelissen");
```

2. Organizing highfrequency data

Users that have their own system set up to store their highfrequency data as `xts` objects can skip this section. The functions to calculate liquidity, etc. in the `highfrequency` package can take highfrequency data of various data vendors as input: NYSE TAQ, WRDS TAQ, Reuters, Bloomberg, etc. To achieve this flexibility, we rely on the functionality provided by the excellent `quantmod` package (Ryan 2011). Additionally, `highfrequency` provides functionality to convert raw data from several data providers into `xts` objects. The latter is discussed and illustrated in this section.

Highfrequency financial data is typically subdivided into two files, containing information on the trades and quotes respectively. `highfrequency` can currently parse three types of input data into `xts` objects: (i) ".txt" files extracted from the NYSE TAQ database, (ii) ".csv" files extracted from the WRDS database, (iii) ".asc" files from <http://www.tickdata.com>. For each of these input data types, we briefly discuss how the `convert` function can be used to convert the txt/csv/asc files into `xts` objects (Ryan and Ulrich 2009) that are saved on-disk in the "RData" format. We opt to store the data as `xts` objects because this type of object can be indexed by an indicator of time and date.¹

Parsing raw NYSE TAQ data into xts:

Suppose the folder `~/raw_data` on your drive contains the two folders: "2008-01-02" and "2008-01-03". Suppose these folders each contain the files `AAPL_trades.txt` and `AA_trades.txt` with the daily TAQ data bought from the New York Stock Exchange (NYSE) for the AAPL and AA stock. The raw data can then easily be converted into on-disk `xts` objects as follows:

```
from = "2008-01-02";
to = "2008-01-03";
datasource = "~/raw_data";
datadestination = "~/xts_data";

convert(from, to, datasource, datadestination, trades=TRUE,
        quotes=FALSE, ticker=c("AA", "AAPL"), dir=TRUE, extension="txt",
```

¹Examples of the use of `xts` objects can be found on <http://www.quantmod.com/examples/data/>.

```
header=FALSE,tradecolnames=NULL,quotecolnames=NULL,
format="%Y%m%d %H:%M:%S");
```

At this point, the folder `~/xts_data` will contain two folders named `2008-01-02` and `2008-01-03` containing the files `AAPL_trades.RData` and `AAPL_quotes.RData` which contain the respective daily trade data. The data can now be loaded with the `TAQLoad` function (see below).

Parsing raw WRDS TAQ data into xts:

Suppose the folder `~/raw_data` on your drive contains the files `IBM_quotes.csv` and `IBM_trades.csv` acquired from Wharton Research Data Service (WRDS).² Both files contain respectively the trade and quote data for the "IBM" stock for "2011-12-01" up to "2011-12-02". The data can then easily be converted into on-disk xts objects as follows:

```
from = "2011-12-01";
to = "2011-12-02";
datasource = "~/raw_data";
datadestination = "~/xts_data";
convert( from=from, to=to, datasource=datasource,
         datadestination=datadestination, trades = T, quotes = T,
         ticker="IBM", dir = TRUE, extension = "csv",
         header = TRUE, tradecolnames = NULL, quotecolnames = NULL,
         format="%Y%m%d %H:%M:%S", onefile = TRUE )
```

Now, the folder `~/xts_data` contains two folders, one for each day in the sample, i.e. "2011-12-01" up to "2011-12-02". Each of these folders contains a file named `IBM_trades.RData` in which the trades for that day can be found and a file `IBM_quotes.RData` in which the quotes for that day can be found, both saved as xts objects. As a final step, the function `TAQLoad` can be used to load the on-disk data into your R workspace. You can for example get the trades for this sample in your workspace as follows:

```
> xts_data = TAQLoad( tickers="IBM", from="2011-12-01",
+                   to="2011-12-02",trades=F,
+                   quotes=TRUE, datasource=datadestination)
> head(xts_data)
```

		SYMBOL	EX	BID	BIDSIZ	OFR	OFRSIZ	MODE
2011-12-01	04:00:00	"IBM"	"P"	"176.85"	"1"	"188.00"	" 1"	"12"
2011-12-01	04:00:17	"IBM"	"P"	"185.92"	"1"	"187.74"	" 1"	"12"
2011-12-01	04:00:18	"IBM"	"P"	"176.85"	"1"	"187.74"	" 1"	"12"
2011-12-01	04:00:25	"IBM"	"P"	"176.85"	"1"	"187.73"	" 1"	"12"
2011-12-01	04:00:26	"IBM"	"P"	"176.85"	"1"	"188.00"	" 1"	"12"
2011-12-01	04:00:26	"IBM"	"P"	"176.85"	"1"	"187.74"	" 1"	"12"

Parsing raw Tickdata.com data into xts:

Suppose the folder `~/raw_data` on your drive contains the files `GLP_quotes.asc` and

²<http://wrds-web.wharton.upenn.edu/wrds>

GLP_trades.asc acquired from www.tickdata.com. Both files contain respectively the trade and quote data for the "GLP" stock for "2011-01-11" up to "2011-03-11". The data can then easily be converted into on-disk xts objects as follows:

```
from = "2011-01-11";
to    = "2011-03-11";
datasource = "~/raw_data";
datadestination = "~/xts_data";
convert(from=from, to=to, datasource=datasource,
        datadestination=datadestination, trades = TRUE,
        quotes = TRUE, ticker="GLP", dir = TRUE, format = "%d/%m/%Y %H:%M:%OS",
        extension = "tickdatacom", header = TRUE, onefile = TRUE );
```

At this point, the folder ~/xts_data now contains three folders, one for each day in the sample, i.e. "2011-01-11", "2011-02-11", "2011-03-11". Each of these folders contains a file named GLP_trades.RData in which the trades for that day can be found and a file GLP_quotes.RData in which the quotes for that day can be found, both saved as xts object. Notice the format = '%d/%m/%Y %H:%M:%OS' argument that allows for timestamps more accurate than seconds, e.g. milliseconds in this case. As a final step, the function TAQLoad can be used to load the on-disk data into your R workspace. You can for example get the trade data for the first day of the sample as follows:

```
> options("digits.secs"=3); #Show milliseconds
> xts_data = TAQLoad(tickers="GLP", from="2011-01-11", to="2011-01-11",
+                 trades=T, quotes=F,
+                 datasource=datadestination)
> head(xts_data)
```

			SYMBOL	EX	PRICE	SIZE	COND	CORR	G127
2011-01-11	09:30:00.338	"GLP"	"T"	"18.0700"	" 500"	"0 X"	"0"	""	
2011-01-11	09:30:00.338	"GLP"	"T"	"18.0700"	" 500"	"Q"	"0"	""	
2011-01-11	09:33:49.342	"GLP"	"T"	"18.5000"	" 150"	"F"	"0"	""	
2011-01-11	09:39:29.280	"GLP"	"N"	"19.2000"	"4924"	"0"	"0"	""	
2011-01-11	09:39:29.348	"GLP"	"D"	"19.2400"	" 500"	"@"	"0"	"T"	
2011-01-11	09:39:29.411	"GLP"	"N"	"19.2400"	" 200"	"F"	"0"	""	

Typical trades and quotes: data description:

Both trades and quotes data should thus be xts objects (Ryan and Ulrich 2009), if you want to use the functionality of highfrequency. Table 1 reports the information (i.e. columns) typically found in trades and quotes data objects.

3. Manipulation of highfrequency data

In this section we discuss two very common first steps in the manipulation of highfrequency financial data: (i) the cleaning and (ii) aggregation of the data.

3.1. Cleaning of highfrequency data

For various reasons, raw trade and quote data contains numerous data errors. Therefore, the

Table 1: Elements of trade and quote data

All data	
SYMBOL	The stock's ticker
EX	Exchange on which the trade/quote occurred
Trade data	
PRICE	Transaction price
SIZE	Number of shares traded
COND	Sales condition code
CORR	Correction indicator
G127	Combined "G", Rule 127, and stopped stock trade
Quote data	
BID	Bid price
BIDSIZ	Bid size in number of round lots (100 share units)
OFR	Offer price
OFRSIZ	Offer size in number of round lots (100 share units)
MODE	Quote condition indicator

data is not suited for analysis right-away, and data-cleaning is an essential step in dealing with tick-by-tick data (Brownlees and Gallo 2006). `highfrequency` implements the step-by-step cleaning procedure proposed by Barndorff-Nielsen *et al.* (2008). Table 2 provides an overview of the cleaning functions. A user can either use a specific cleaning procedure or a wrapper function that performs multiple cleaning steps. The wrapper functions offer on-disk functionality: i.e. load on-disk raw data and save the clean data back to your hard disk. This method is advisable in case you have multiple days of data to clean. Of course, the data on your disk should be organized as discussed in the previous section to benefit from this functionality. More specifically, these functions expect that the data is saved as xts objects saved in a folder for each day (as the `TAQLoad` function does), which will always be the case if you used the `convert` function to convert the raw data into xts objects.

To maintain some insight in the cleaning process, the functions `tradesCleanup` and `quotesCleanup` report the total number of remaining observations after each cleaning step:

```
> data("sample_tdataraw");
> dim(sample_tdataraw);
[1] 48484      7
> tdata_afterfirstcleaning = tradesCleanup(tdataraw=sample_tdataraw,exchanges="N");
> tdata_afterfirstcleaning$report;
      initial number      no zero prices      select exchange
           48484           48479           20795
      sales condition merge same timestamp
           20135           9105
> dim(tdata_afterfirstcleaning$tdata)
[1] 9105      7
```

Table 2: Cleaning functions

Function	Function Description
All Data:	
ExchangeHoursOnly	Restrict data to exchange hours
selectexchange	Restrict data to specific exchange
Trade Data:	
noZeroPrices	Delete entries with zero prices
autoSelectExchangeTrades	Restrict data to exchange with highest trade volume
salesCondition	Delete entries with abnormal Sale Condition
mergeTradesSameTimestamp	Delete entries with same time stamp and use median price
rmTradeOutliers	Delete entries with prices above/below ask/bid +/- bid/ask spread
Quote Data:	
noZeroQuotes	Delete entries with zero quotes
autoSelectExchangeQuotes	Restrict data to exchange with highest bidsize + offersize
mergeQuotesSameTimestamp	Delete entries with same time stamp and use median quotes
rmNegativeSpread	Delete entries with negative spreads
rmLargeSpread	Delete entries if spread > maxi*median daily spread
rmOutliers	Delete entries for which the mid-quote is outlying with respect to surrounding entries
Wrapper cleanup functions (perform sequentially the following for on-disk data)	
tradesCleanup	noZeroPrices, selectExchange, salesCondition, mergeTradesSameTimestamp.
quotesCleanup	noZeroQuotes, selectExchange, rmLargeSpread, mergeQuotesSameTimestamp rmOutliers
tradesCleanupFinal	rmTradeOutliers (based on cleaned quote data as well)

3.2. Aggregation of highfrequency data

Prices are typically not recorded at equispaced time points, while e.g. many realized volatility measures rely on equispaced returns. Furthermore, prices are often observed at different points in time for different assets, while e.g. most multivariate realized volatility estimators rely on synchronized data (see e.g. Table 3). There several ways to force these asynchronously and/or irregularly recorded series to a synchronized and/or equispaced time grid.

The most popular method, previous tick aggregation, forces prices to an equispaced grid by taking the last price realized before each grid point. `highfrequency` provides users with the `aggregatets` function for fast and easy previous tick aggregation:

```
> library("highfrequency");
> # Load sample price data
> data("sample_tdata");
> ts = sample_tdata$PRICE;
> # Previous tick aggregation to the 5-minute sampling frequency:
> tsagg5min = aggregatets(ts,on="minutes",k=5);
> head(tsagg5min);
```

```
                PRICE
2008-01-04 09:35:00 193.920
2008-01-04 09:40:00 194.630
2008-01-04 09:45:00 193.520
2008-01-04 09:50:00 192.850
```

```
2008-01-04 09:55:00 190.795
2008-01-04 10:00:00 190.420
```

```
> # Previous tick aggregation to the 30-second sampling frequency:
> tsagg30sec = aggregatets(ts,on="seconds",k=30);
> tail(tsagg30sec);
```

```
                PRICE
2008-01-04 15:57:30 191.790
2008-01-04 15:58:00 191.740
2008-01-04 15:58:30 191.760
2008-01-04 15:59:00 191.470
2008-01-04 15:59:30 191.825
2008-01-04 16:00:00 191.670
```

In the example above the prices are forced to a regular equispaced time grid of respectively 5 minutes and 30 seconds. Furthermore, the `aggregatets` function is build into all realized measures (see Section 4) and can be called by setting the arguments `align.by` and `align.period`. In that case, first the price(s) are forced the an equispaced regular time grid and then the realized measure is calculated based on the returns over these regular time periods to which the observations were forced. This has the advantage that the user can input the original price series into the realized measures without having to worry about the asynchronicity or the irregularity of the price series.

Another synchronization method (not integrated into the realized measures) is refresh time, initially proposed by [Harris and Wood \(1995\)](#) and recently advocated in [Barndorff-Nielsen et al. \(2011\)](#). The function `refreshTime` in `highfrequency` can be used to force time series (very fast) to a synchronized but not necessarily equispaced time grid. The so-called refresh times are the time points at which all assets have traded at least once since the last refresh point. More specifically, the first refresh time corresponds to the first time at which all stocks have traded. The subsequent refresh time is defined as the first time when all stocks have been traded again. This process is repeated until the end of one time series is reached.

Illustration on price aggregation with refresh time and subsequent volatility calculation:

```
> data("sample_tdata");
> data("sample_qdata");
> #We assume that stock1 and stock2 contain price data on imaginary stocks:
> stock1 = sample_tdata$PRICE;
> stock2 = sample_qdata$BID;
> #Previous-tick aggregation to one minute:
> mPrice_1min = cbind(aggregatePrice(stock1),aggregatePrice(stock2));
> #Refresh time aggregation:
> mPrice_Refresh = refreshTime(list(stock1,stock2));
> #Calculate a jump robust volatility measures
> #based on synchronized data:
> rbpcov1 = rBPCov(mPrice_1min,makeReturns=TRUE);
> rbpcov2 = rBPCov(mPrice_Refresh,makeReturns=TRUE);
```

```
> #Calculate a jump and microstructure noise robust volatility measure  
> #based on nonsynchronous data:  
> rtscov = rTSCov(list(stock1,stock2));
```

4. Realized volatility measures

The availability of high-frequency data has enabled researchers to estimate the ex post realized volatility based on squared intraday returns (Andersen *et al.* 2003). In practice, the main challenges in univariate volatility estimation are dealing with (i) jumps in the price level and (ii) microstructure noise. Multivariate volatility estimation is additionally challenging because of (i) the asynchronicity of observations between assets and (ii) the need for a positive semidefinite covariance matrix estimator. The `highfrequency` package implements many recently proposed realized volatility and covolatility measures³

An overview of the univariate and multivariate volatility estimators implemented in `highfrequency` is given in Table 3. The first two columns indicate whether the estimator can be applied to univariate or multivariate price series. The following two columns indicate whether the estimator is robust with respect to jumps and microstructure noise respectively. The next column reports whether asynchronous price series can/should be used as input. The last column indicates whether the estimator always yields a positive semidefinite matrix in the multivariate case. All realized measures have (at least) the following arguments:

- `rdata`: The return data.
 - In the univariate case: an `xts` object containing the (tick) data for one day.
 - In the multivariate case:
 - * In case of synchronized observations: a (M x N) matrix/`zoo`/`xts` object containing the N return series over period t, with M observations during t.
 - * In the case of asynchronous observations: a list. Each list-item i contains an `xts` object with the intraday data of stock i for day t.
- `align.by`: a string, align the tick data to "seconds"|"minutes"|"hours".
- `align.period`: an integer, align the tick data to this many [seconds|minutes|hours].
- `makeReturns`: boolean, should be TRUE when `rdata` contains prices instead of returns. FALSE by default.
- Arguments relevant for multivariate case (i.e. realized covolatility measures):
 - `cor`: boolean, in case it is TRUE, the correlation is returned. FALSE by default.
 - `makePsd`: boolean. Non positive semidefinite estimates can be transformed into a positive semidefinite matrix by setting this argument to TRUE. The eigenvalue method is used.
- ...: additional arguments depending on the type of realized measure.

Another interesting feature of high-frequency data in the context of volatility measurement is the periodicity in the volatility of high-frequency returns induced by opening, lunch and closing of financial markets (Andersen and Bollerslev 1997). `highfrequency` implements both

³ Throughout we use the terms realized volatility and realized variance interchangeably to denote high-frequency data based estimators for the daily variation in the returns. All outputted values should be interpreted as variances, unless they clearly indicate standard deviations, e.g. in the `spotVol` function.

Table 3: Overview of Volatility estimators

Estimator	Univariate	Multivariate	Jump robust	Microstructure noise robust	Tick-by-tick returns as input	Positive semidefinite
medRV (Andersen <i>et al.</i> 2012)	x		x			/
minRV (Andersen <i>et al.</i> 2012)	x		x			/
rCov (Andersen <i>et al.</i> 2003)	x	x				x
rBPCov (Barndorff-Nielsen and Shephard 2004)	x	x	x			
rOWCov (Boudt <i>et al.</i> 2011a)	x	x	x			x
rThresholdCov (Gobbi and Mancini 2009)		x	x			x
rTSCov (Zhang 2011)	x	x		x	x	
rRTSCov (Boudt and Zhang 2010)	x	x	x	x	x	
rAVGCov (Ait-Sahalia <i>et al.</i> 2005)	x	x		x	x	x
rKernelCov (Barndorff-Nielsen <i>et al.</i> 2004)	x	x		x	x	x
rHYCov (Hayashi and Yoshida 2005)		x			x	

the classic intraday periodicity estimation method of Andersen and Bollerslev (1997) and a jump robust version proposed by Boudt *et al.* (2011b). These estimation methods assume that the standard deviation of intraday returns (called `vol` in output) can be decomposed in a daily volatility factor (constant for all intraday returns observed on the same day, called `dailyvol`) and an intraday factor (deterministic function of the intraday time, called `periodicvol`). The estimates are either non-parametric (based on a scale estimator) or parametric (based on regression estimation of a flexible specification for the intraday factor).

The sample code below illustrates the estimation of intraday periodicity as well as Figure 1:

```
> data("sample_real5minprices");
>
> #compute and plot intraday periodicity
> out = spotVol(sample_real5minprices,P1=6,P2=4,periodicvol="TML",k=5,
+ dummies=FALSE);
> head(out);
```

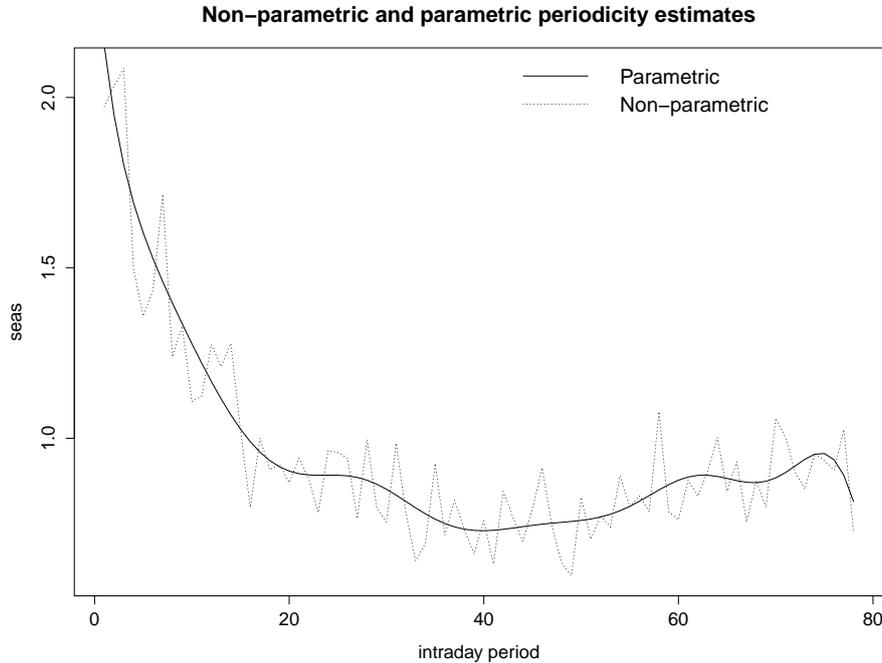
	returns	vol	dailyvol	periodicvol
2005-03-04 09:35:00	-0.0010966963	0.004081072	0.001896816	2.151539
2005-03-04 09:40:00	-0.0005614217	0.003695715	0.001896816	1.948379
2005-03-04 09:45:00	-0.0026443880	0.003417950	0.001896816	1.801941

5. Volatility forecasting

It is broadly accepted by academic researchers that, when appropriately managed, access to high-frequency data leads to a comparative advantage in better predicting the volatility of future price evolution and hence the generation of alpha through quantitative investment models. Already in 2003 Fleming *et al.* (2003) estimated that an investor would be willing to pay 50 to 200 basis points per year to capture the gains in portfolio performance, from using high-frequency returns instead of daily returns for volatility forecasting. His findings were later confirmed in a somewhat different setting by the research of De Pooter *et al.* (2008).

In this section we discuss 2 classes of univariate forecasting models implemented in the `highfrequency` package that have highfrequency data as input: (i) the Heterogeneous Autoregressive model for Realized Volatility (HAR) discussed in Andersen *et al.* (2007) and

Figure 1: Estimated periodicity pattern using spotVol function.



Corsi (2009a) and (ii) the HEAVY model (High frEQUENCY bAsed VolatilitY) introduced in Shephard and Sheppard (2010). The goal of both models is roughly speaking to predict the next days volatility based on the highfrequency price evolutions prior to that day.

Although the HAR and the HEAVY model have the same goal, i.e. modeling the conditional volatility, they take a different approach. Whereas the HAR model is focused on predicting the open-to-close variation, the HEAVY model has two equations: one focused on the open-to-close variation, one focused on the close-to-close variation. The main advantages of the HAR model are among other things, that it is simple and easy to estimate (since it is essentially a special type of linear model that can be estimated by least squares), even though it still manages to reproduce typical features found in volatility such as the long memory, fat tails, etc. The main advantages of the HEAVY model are among other things, that it models both the close-to-close conditional variance, as well as the conditional expectation of the open-to-close variation. Furthermore, the HEAVY model has momentum and mean reversion effects, and it adjusts quickly to structural breaks in the level of the volatility process. In contrast to the HAR model, the estimation of the HEAVY model is done by Gaussian quasi-maximum likelihood, for both equations in the model separately.

The next two sections review the HAR model and HEAVY model each in more detail, and of course discuss and illustrate how `highfrequency` can be used to estimate these models.

5.1. The HAR model

Corsi (2009b) proposed the Heterogeneous Autoregressive (HAR) model for Realized Volatil-

ity. As a first step, intraday returns are used to measure the daily volatility using e.g. simply Realized Volatility and typically ignoring the overnight return. As a second step, the realized volatility is parameterized as a linear function of the lagged realized volatilities over different horizons. Typically, the daily, weekly and monthly horizons are chosen to aggregate the realized volatility over. The simplest version of the model, as proposed in Corsi (2009b), is referred to as the HAR-RV model. In an interesting paper, Andersen *et al.* (2007) extend the model of Corsi (2009b) by explicitly including the contribution of jumps to the daily Realized Volatility in the model. Their analysis suggests that the volatility originating from jumps in the price level is less persistent than the volatility originating from the continuous component of Realized Volatility, and that separating the rough jump moves from the smooth continuous moves enables better forecasting results. They propose mainly two ways to model this idea, and both are implemented in the `highfrequency` package. In what follows, we first discuss the different versions of the HAR model in more detail, and then we discuss and illustrate how `highfrequency` can be used to estimate them.

The `harModel` function in `highfrequency` implements the Heterogeneous Autoregressive model for Realized Volatility discussed in Andersen *et al.* (2007) and Corsi (2009a). Let $r_{t,i}$ the i -th intraday return on day t , for $i = 1, \dots, M$ with M the number of intraday returns per day. The realized volatility is then given by $RV_t = \sum_{i=1}^M r_{t,i}^2$. Denote by

$$RV_{t,t+h} = h^{-1}[RV_{t+1} + RV_{t+2} + \dots + RV_{t+h}],$$

the Realized volatility aggregate over h days. Note that by definition $RV_{t,t+1} \equiv RV_{t+1}$.

The HARRV model

The most basic volatility model discussed Andersen *et al.* (2007) and Corsi (2009a) is given by:

$$RV_{t,t+h} = \beta_0 + \beta_D RV_t + \beta_W RV_{t-5,t} + \beta_M RV_{t-22,t} + \epsilon_{t,t+h},$$

for $t = 1, 2, \dots, T$. Typically $h = 1$, and the model simplifies to

$$RV_{t+1} = \beta_0 + \beta_D RV_t + \beta_W RV_{t-5,t} + \beta_M RV_{t-22,t} + \epsilon_{t+1}.$$

The HARRVJ model

In the presence of jumps in intraday price process, the traditional realized volatility measures are dominated by the contribution of the jumps to volatility. Therefore, robust measure of volatility were developed in order to estimate only the "smooth" price variation (roughly speaking). The contribution of the jumps to the quadratic price process can then be estimated by $J_t = \max[RV_t - BPV_t, 0]$. The HAR-RV-J model is then given by

$$RV_{t,t+h} = \beta_0 + \beta_D RV_t + \beta_W RV_{t-5,t} + \beta_M RV_{t-22,t} + J_t + \epsilon_{t,t+h}.$$

The HARRVCJ model

Andersen *et al.* (2007) argue that it may be desirable to treat small jumps as measurement errors, or part of the continuous sample path variation process, associating only large values

of $(RV_t - BPV_t)$ with the jump component. Therefore, they define Z_t as

$$Z_t = (1/n)^{-1/2} \times \frac{(RV_t - BPV_t)RV_t^{-1}}{[(\mu_1^{-4} + 2\mu_1^{-2} - 5) \max\{1, TQ_t BPV_t^{-2}\}]^{1/2}}, \quad (1)$$

where $TQ_t = M \times \mu_{4/3}^{-3} \sum_{j=3}^M |r_{t,j}|^{4/3} |r_{t,(j-1)}|^{4/3} |r_{t,(j-2)}|^{4/3}$, with $\mu_p = E(|z|^p)$. Suppose you identify the "significant" jumps by the realizations of Z_t in excess of some critical value, say Φ_α ,

$$J_t = I[Z_t < \Phi_\alpha]RV_t + I[Z_t \leq \Phi_\alpha]BPV_t,$$

where $I[\cdot]$ denotes the indicator function. To ensure that the estimated continuous sample path component variation and jump variation sum to the total realized variation, we follow Andersen *et al.* (2007) in estimating the former component as the residual,

$$C_t = I[Z_t \leq \Phi_\alpha]RV_t + I[Z_t > \Phi_\alpha]BPV_t.$$

The HAR-RV-CJ model is then given by:

$$RV_{t,t+h} = \beta_0 + \beta_{CD}C_t + \beta_{CW}C_{t-5,t} + \beta_{CM}C_{t-22,t} + J_t + \epsilon_{t,t+h}.$$

Usage and arguments

The `harModel` function in `highfrequency` takes as `data` input an `xts` object containing the intraday returns. You can specify the type of HAR model that you want to estimate by setting the `type` argument. By default, the most basic model is chosen `type='HARRV'`. Other valid options are `type='HARRVJ'` or `type='HARRVCJ'` (see above for a discussion of these models).

Based on the intraday returns, daily realized measures are calculated. With the argument `RVest`, you can set which volatility estimators should be used to estimate both the daily integrated variance (non-jump-robust) and the continuous component of volatility. By default, total daily volatility is measured by Realized Volatility and the continuous component is measured by the Realized Bipower Variation, hence `RVest=c(rCov, rBPCov)`, but users can set estimators to their liking (see Section 4 for a discussion on the implemented Realized Volatility measures).

The daily volatility measures are subsequently aggregated over different time horizons. Use the arguments to `periods` and `periodsJ` to specify the horizons over which the continuous/total volatility and the jump component should be aggregated. Typically, the daily, weekly and monthly frequency are used, i.e. one, five and twenty-two days which translates into the default setting: `periods=c(1,5,22)` and `periodsJ=c(1,5,22)`.

Corsi and Reno (2012) propose to further extend the HAR model with a leverage component (Leverage Heterogeneous Auto-Regressive model). This allows to take into account that negative returns affect future volatility differently than positive returns. By default, the leverage effect is not taken into account and the argument `leverage=NULL`. In case you want to mimic the analysis in Corsi and Reno (2012), with leverage components aggregated on the daily, weekly and monthly frequency, just set `leverage=c(1,5,22)`.

In the `HARRVCJ` a test determines whether the contribution of the jumps is statistically significant. The argument `jumpstest` should be set to the function name of the test to determine whether the jump variability is significant that day. By default `jumpstest='ABDJumpstest'`,

hence using the test statistic in Equation (1) (or Equation (18) of Andersen *et al.* (2007)). The argument `alpha` can then be used to set the confidence level used in testing for jumps, which is set to the traditional "5%" by default.

Often, the object will be to model the volatility for the next day, but this needn't be the case. Use the argument `h` to set the time horizon over which you want to aggregate the dependent variable, e.g. `h=5` in case you are interested in modeling the weekly realized volatility.

To conclude, we note that in applications of the HAR model sometimes the dependent and explanatory variables are transformed by taking the logarithm or the square root. Set the argument `transform='log'` or `transform='sqrt'` or any other function to transform all variables right before the linear model is estimated. By default, no transformation is done and `transform=NULL`.

Examples

Fitting the HARRV model on the Dow Jones Industrial Average in 2008

As a first step, we load the daily realized volatility for the Dow Jones Industrial average and select only 2008. The sample dataset `realized_library` in `highfrequency` contains a very rich set of realized volatility measures computed by Heber *et al.* (2009). `highfrequency` only contains a subset of the full realized library, which can be found on their website: <http://realized.oxford-man.ox.ac.uk>.

```
> data(realized_library); #Get sample daily Realized Volatility data
> DJI_RV = realized_library$Dow.Jones.Industrials.Realized.Variance; #Select DJI
> DJI_RV = DJI_RV[!is.na(DJI_RV)]; #Remove NA's
> DJI_RV = DJI_RV['2008'];
```

As a second step, we compute the traditional Heterogeneous AutoRegressive model (HAR). The output of the model is an S3 object. Since the HAR-model is simply a special type of linear model, it is also implemented that way: the output of the `harModel` function is a subclass `harModel` of `lm`, the standard class for linear models. Figure 2 plots the output object of the `harModel` function, which has time on the horizontal axis and the observed realized volatility and the forecasted realized volatility on the vertical axis (of course this analysis is in-sample, but the estimated coefficients of the model can be used for real out-of-sample forecasts obviously). It is clear from a visual inspection of Figure 2 that one of the features of the `harModel` is that it can adapt relatively fast to changes in the volatility level, which explains its popularity in recent years.

```
> x = harModel(data=DJI_RV , periods = c(1,5,22), RVest = c("rCov"),
+             type="HARRV",h=1,transform=NULL);
> class(x);
```

```
[1] "harModel" "lm"
```

```
> x;
```

```
Model:
```

```
RV1 = beta0 + beta1 * RV1 + beta2 * RV5 + beta3 * RV22
```

Coefficients:

beta0	beta1	beta2	beta3
4.432e-05	1.586e-01	6.213e-01	8.721e-02

r.squared	adj.r.squared
0.4679	0.4608

> summary(x);

Call:

"RV1 = beta0 + beta1 * RV1 + beta2 * RV5 + beta3 * RV22"

Residuals:

Min	1Q	Median	3Q	Max
-0.0017683	-0.0000626	-0.0000427	-0.0000087	0.0044331

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
beta0	4.432e-05	3.695e-05	1.200	0.2315
beta1	1.586e-01	8.089e-02	1.960	0.0512 .
beta2	6.213e-01	1.362e-01	4.560	8.36e-06 ***
beta3	8.721e-02	1.217e-01	0.716	0.4745

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.0004344 on 227 degrees of freedom

Multiple R-squared: 0.4679, Adjusted R-squared: 0.4608

F-statistic: 66.53 on 3 and 227 DF, p-value: < 2.2e-16

> plot(x);

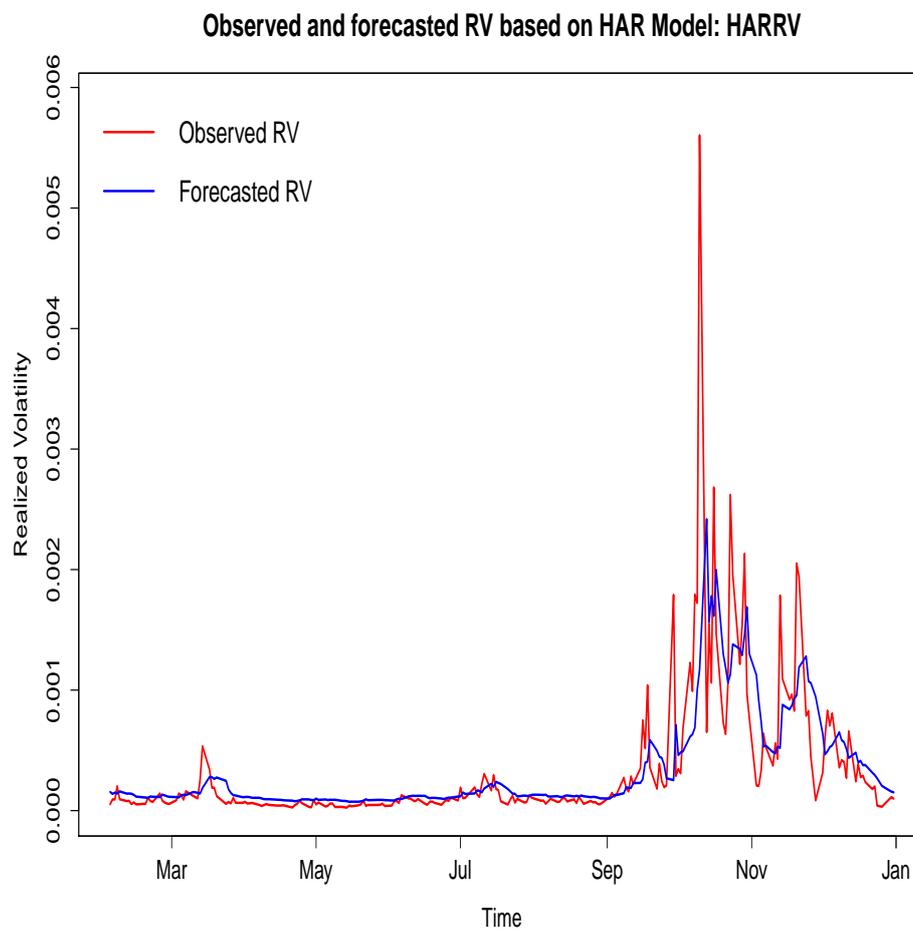
Fitting the HARRVCJ model on small example dataset

Using `highfrequency`, it is also very easy to estimate more sophisticated versions of the `harModel`. For example, the HARRVCJ model discussed in [Andersen *et al.* \(2007\)](#) can be easily estimated with a small example dataset as follows:

```
> data("sample_5minprices_jumps");
> data = sample_5minprices_jumps[,1];
> data = makeReturns(data); #Get the high-frequency return data
> x      = harModel(data, periods = c(1,5,10), periodsJ=c(1,5,10),
+          RVest = c("rCov","rBPCov"), type="HARRVCJ",
+          transform="sqrt");
> x
```

Model:

Figure 2: HAR-model for the Realized Volatility of the DJIA in 2008



```
sqrt(RV1) = beta0 + beta1 * sqrt(C1) + beta2 * sqrt(C5) + beta3 * sqrt(C10)
          + beta4 * sqrt(J1) + beta5 * sqrt(J5) + beta6 * sqrt(J10)
```

Coefficients:

```
beta0    beta1    beta2    beta3    beta4    beta5
-0.8835  1.1957  -25.1922  38.9909  -0.4483  0.8084
beta6
-6.8305
```

```
r.squared  adj.r.squared
0.9915     0.9661
```

This last example shows how easily a special type of HAR model can be estimated having solely the intraday returns as input. Note that this last example purely serves a didactic purpose: the high R-squared can be explained by the small number of observations (i.e. days) in the dataset.

5.2. The HEAVY model

Shephard and Sheppard (2010) introduced the HEAVY model (High frEQUENCY bAsed Volatility) to leverage highfrequency data for volatility modeling. Essentially, the model boils down to two equations: Equation (2) models the close-to-close conditional variance, while Equation (3) models the conditional expectation of the open-to-close variation. Denote by \mathcal{F}_{t-1}^{HF} the past high frequency data (i.e. all intraday returns) and by \mathcal{F}_{t-1}^{LF} the past low frequency data (i.e. daily returns). The HEAVY model is now based on \mathcal{F}_{t-1}^{HF} in contrast to traditional GARCH models that use only \mathcal{F}_{t-1}^{LF} .

Denote the daily returns:

$$r_1, r_2, \dots, r_T.$$

Based on intraday data, one can calculate the daily realized measures:

$$RM_1, RM_2, \dots, RM_T,$$

with T the total number of days in the sample. The heavy model is given by:

$$\text{Var}(r_t | \mathcal{F}_{t-1}^{HF}) = h_t = \omega + \alpha RM_{t-1} + \beta h_{t-1}, \quad \omega, \alpha \geq 0 \text{ and } \beta \in [0, 1], \quad (2)$$

$$\text{E}(RM_t | \mathcal{F}_{t-1}^{HF}) = \theta_t = \omega_R + \alpha_R RM_{t-1} + \beta_R \theta_{t-1}, \quad \omega_R, \alpha_R, \beta_R \geq 0 \text{ and } \alpha_R + \beta_R \in [0, 1]. \quad (3)$$

Equation (2) models the close-to-close conditional variance, and equation (3) models the conditional expectation of the open-to-close variation. In matrix notation, the model becomes:

$$\begin{pmatrix} h_t \\ \theta_t \end{pmatrix} = \begin{pmatrix} \omega \\ \omega_R \end{pmatrix} + \begin{pmatrix} 0 & \alpha \\ 0 & \alpha_R \end{pmatrix} \begin{pmatrix} r_{t-1}^2 \\ RM_{t-1} \end{pmatrix} + \begin{pmatrix} \beta & 0 \\ 0 & \beta_R \end{pmatrix} \begin{pmatrix} h_{t-1} \\ \theta_{t-1} \end{pmatrix}. \quad (4)$$

The function `heavyModel` in the `highfrequency` package implements the above model in very general way. It enables the user to obtain the estimates for the parameters $(\omega, \omega_R, \alpha; \alpha_R, \beta, \beta_R)$ using Quasi-maximum likelihood.⁴

⁴The implementation is (loosely) based on the matlab code from Kevin Sheppard: http://www.kevinshppard.com/wiki/MFE_Toolbox

Usage and arguments

The `heavyModel`⁵ takes a $(T \times K)$ matrix containing the data as input, with T the number of days. For the traditional HEAVY model: $K = 2$, the first column contains the squared demeaned daily returns, i.e. r_1^2, \dots, r_T^2 and the second column contains the realized measures, i.e. RM_1, \dots, RM_T .

As you can see in Equation (4), the matrix notation of the HEAVY model contains two matrices with parameters named α and named β respectively. This matrix structure allows to see that the traditional HEAVY model given in (4) is just a special case of a larger set of models that can be written this way. You can use the arguments `p` and `q` from the `heavyModel` function to specify for each of these two matrices: (i) which parameters in the matrix should be estimated by setting a non-zero integer in that matrix cell (ii) how many lags should be included in the model. More formally: `p` is a $(K \times K)$ matrix containing the lag length for the model innovations. Position (i, j) in the matrix indicates the number of lags in equation i of the model for the innovations in data column j . For the traditional heavy model introduced above `p` is given by (cfr. the matrix with the α s in Equation (4)):

$$p = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}.$$

Similarly to the matrix `p`, the matrix `q` is a $(K \times K)$ matrix containing the lag length for the conditional variances. Position (i, j) in the matrix indicates the number of lags in equation i of the model for conditional variances corresponding to series j . For the traditional heavy model introduced above `q` is given by (cfr. the matrix with the β s in Equation (4)):

$$q = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

The starting values that will be used in the optimization can be set by the argument `startingvalues`. The arguments `LB` and `UB` can be used to set the Upper and Lower Bound for the parameters. By default, the lower bound is set to zero and the upperbound to infinity. To specify how the estimation should be initialized you can use the `backcast` argument, which will be set to the unconditional estimates by default. Finally, the `comconst` argument is a boolean indicating how the ω s should be estimated. In case it is `TRUE`, ω s are estimated in the optimization, and in case it is `FALSE` volatility targeting is done and ω is just 1 minus the sum of all relevant α s and β s multiplied by the unconditional variance.

Output

The output of the `heavyModel` is a list. Most interestingly, the list-item `estparams` contains a matrix with the parameter estimates and their names. The order in which the parameters are reported is as follows: first the estimates for ω , then the estimates for the non-zero α s with the most recent lags first in case $\max(p) > 1$, then the estimates for the non-zero β s with the most recent lag first in case $\max(q) > 1$.

Whereas the `loglikelihood` list-item reports the total log likelihood, we offer users a bit more insight with the list-item `likelihoods` containing an `xts`-object with the daily log likelihood

⁵The implementation of the `heavyModel` is not completely finished. For the moment only bound constraints on the parameters are imposed in the optimization. Future developments also include outputting standard errors, and a c-implementation of the likelihood function to speed up the QML estimation.

evaluated at the parameters. The list-item `convergence` provides more information on the convergence of the optimization, see `optim` for more information.

The list-item `condvar` is a $(T \times K)$ xts-object containing the conditional variances. For the traditional HEAVY model, Figure 3 plots for example the conditional close-to-close variance for the DJIA for 1996 up to 2009 (see next subsection for more information).

Examples

Fitting the HEAVY model on the Dow Jones Industrial Average from 1996 up to 2009

As a first step, we load the `realized_library` (Heber *et al.* 2009) which contains information on the Dow Jones Industrial Average. We then select from this library the daily returns and the daily Realized Kernel estimates (Barndorff-Nielsen *et al.* 2004). The data matrix that serves as an input for the `heavyModel` now has the returns as the first column and the Realized Kernel estimates in the second column. We further set the argument `backcast` to the variance of the daily returns and the average realized kernel over the sample period. We now have all ingredients to estimate the HEAVY Model. Based on the output of the model, Figure 3 plots the conditional open-to-close variance, as estimated by the second equation in the model. Note the striking peak in 2008 at the start of the financial crisis.

```
> # Implementation of the heavy model on DJIA:
> data("realized_library");
> returns = realized_library$Dow.Jones.Industrials>Returns;
> rk      = realized_library$Dow.Jones.Industrials.Realized.Kernel;
> returns = returns[!is.na(rk)]; rk = rk[!is.na(rk)]; # Remove NA's
> data    = cbind( returns^2, rk );
> backcast = matrix( c(var(returns),mean(rk)) ,ncol=1);
>
> startvalues = c(0.004,0.02,0.44,0.41,0.74,0.56); # Initial values
> output = heavyModel( data = as.matrix(data,ncol=2), comconst=FALSE,
+                       startingvalues = startvalues, backcast=backcast);
> output$estparams
      [,1]
omega1 0.01750506
omega2 0.06182249
alpha1 0.45118753
alpha2 0.41204541
beta1  0.73834594
beta2  0.56367558
```

6. Liquidity

6.1. Matching trades and quotes

Trades and quotes are often supplied as separate data objects. For many research and practical questions related to transaction data, one needs to merge trades and quotes. Since trades and

Figure 3: HEAVY Model for the Realized Volatility of the DJIA in 2008.

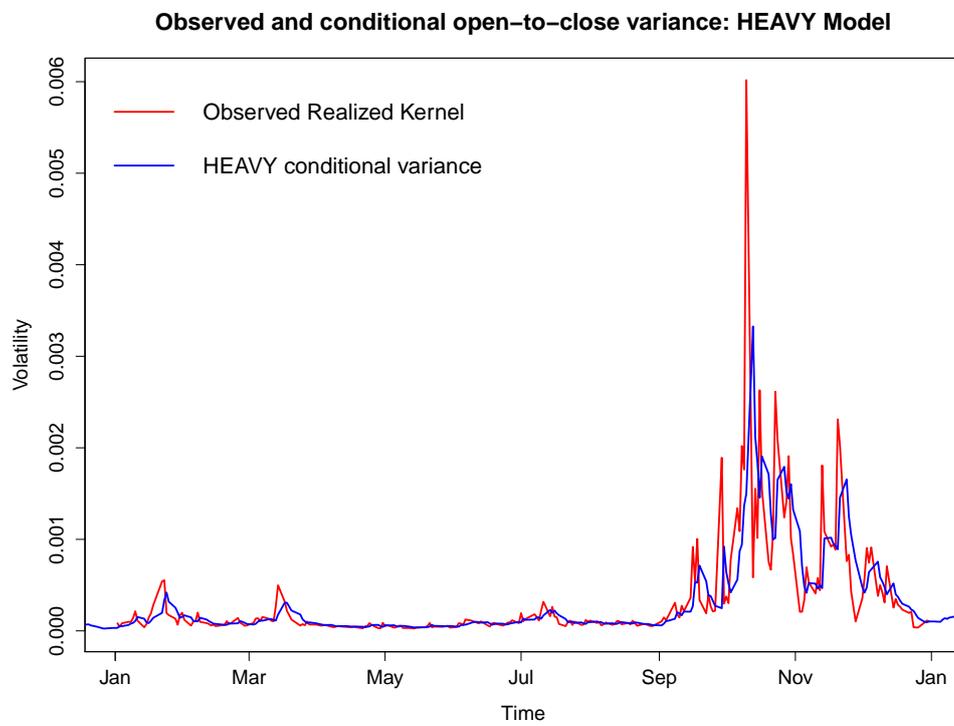


Table 4: Overview of Liquidity measures

Argument(s)	Liquidity measures
es, rs	Compute effective (realized) spread
value_trade	Compute trade value (price \times size)
signed_value_trade	Compute signed trade value
signed_trade_size	Compute signed trade size
di_diff, di_div	Compute depth imbalance
pes, prs	Compute proportional effective and realized spread
price_impact, prop_price_impact	Compute price impact
tspread, pts	Compute half traded and proportional half-traded spread
qs, logqs	Compute quoted spread
qsslope, logqslope	Compute quoted slope

quotes can be subject to different reporting lags, this is not a straightforward operation (Lee and Ready 1991). The function `matchTradesQuotes` can be used for matching trades and quotes. One should supply the number of seconds quotes are registered faster than trades. Based on the research of Vergote (2005), we set 2 seconds as the default.

6.2. Inferred trade direction

Many trades and quotes databases do not indicate whether individual trades are market buy or market sell orders. `highfrequency` implements with `getTradeDirection` the Lee-Ready rule (Lee and Ready 1991) to infer the trade direction based on the matched trades and quotes.

6.3. Liquidity measures

Numerous liquidity measures can be calculated based on matched trade and quote data, using the function `tqLiquidity` (see Bessembinder (2003), Boehmer (2005), Hasbrouck and Seppi (2001) and Venkataraman (2001) for more information on the implemented liquidity measures). The main implemented liquidity measures are listed in Table 4, and can be used as arguments of the function `tqLiquidity`.

The example below illustrates how to: (i) match trades and quotes, (ii) get the trade direction and (iii) calculate liquidity measures.

```
> #Load data samples
> data("sample_tdata");
> data("sample_qdata");
> #Match the trade and quote data
> tqdata = matchTradesQuotes(sample_tdata, sample_qdata);
> #Display information in tqdata
> colnames(tqdata)[1:6];
```

```
[1] "SYMBOL" "EX"      "PRICE"  "SIZE"   "COND"   "CORR"
```

```

> colnames(tqdata)[7:12];

[1] "G127" "BID" "BIDSIZ" "OFR" "OFRSIZ" "MODE"

> #Get the inferred trade direction according to the Lee-Ready rule
> x = getTradeDirection(tqdata);
> #Calculate the proportional realized spread:
> prs = tqLiquidity(tqdata,sample_tdata,sample_qdata,type="prs");
> #Calculate the effective spread:
> es = tqLiquidity(tqdata,type="es");

```

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