

lmer for SAS PROC MIXED Users

Douglas Bates
Department of Statistics
University of Wisconsin – Madison
Bates@wisc.edu

1 Introduction

The `lmer` function from the `Matrix` library for R is used to fit linear mixed-effects models. It is similar in scope to the SAS procedure PROC MIXED described in Littell et al. (1996).

A file on the SAS Institute web site (<http://www.sas.com>) contains all the data sets in the book and all the SAS programs used in Littell et al. (1996). We have converted the data sets from the tabular representation used for SAS PROC MIXED to the `groupedData` objects used by `lmer`. To help users familiar with SAS PROC MIXED get up to speed with `lmer` more quickly, we provide transcripts of some `lmer` analyses paralleling the SAS PROC MIXED analyses in Littell et al. (1996).

In this paper we highlight some of the similarities and differences of `lmer` analysis and SAS PROC MIXED analysis.

2 Similarities between lmer and SAS PROC MIXED

Both SAS PROC MIXED and `lmer` can fit linear mixed-effects models expressed in the Laird-Ware formulation. For a single level of grouping Laird and Ware (1982) write the n_i -dimensional response vector \mathbf{y}_i for the i th experimental

unit as

$$\begin{aligned} \mathbf{y}_i &= \mathbf{X}_i\boldsymbol{\beta} + \mathbf{Z}_i\mathbf{b}_i + \boldsymbol{\epsilon}_i, \quad i = 1, \dots, M \\ \mathbf{b}_i &\sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}), \quad \boldsymbol{\epsilon}_i \sim \mathcal{N}(\mathbf{0}, \sigma^2 \mathbf{I}) \end{aligned} \tag{1}$$

where $\boldsymbol{\beta}$ is the p -dimensional vector of *fixed effects*, \mathbf{b}_i is the q -dimensional vector of *random effects*, \mathbf{X}_i (of size $n_i \times p$) and \mathbf{Z}_i (of size $n_i \times q$) are known fixed-effects and random-effects regressor matrices, and $\boldsymbol{\epsilon}_i$ is the n_i -dimensional *within-group error* vector with a spherical Gaussian distribution. The assumption $\text{Var}(\boldsymbol{\epsilon}_i) = \sigma^2 \mathbf{I}$ can be relaxed using additional arguments in the model fitting.

The basic specification of the model requires a linear model expression for the fixed effects and a linear model expression for the random effects. In SAS PROC MIXED the fixed-effects part is specified in the `model` statement and the random-effects part in the `random` statement. In `lmer` the arguments are called `fixed` and `random`.

Both SAS PROC MIXED and `lmer` allow a mixed-effects model to be fit by maximum likelihood (`method = ml` in SAS) or by maximum residual likelihood, sometimes also called restricted maximum likelihood or REML. This is the default criterion in SAS PROC MIXED and in `lmer`. To get ML estimates in `lmer`, set the optional argument `method="REML"`.

3 Important differences

The output from PROC MIXED typically includes values of the Akaike Information Criterion (AIC) and Schwartz’s Bayesian Criterion (SBC). These are used to compare different models fit to the same data. The output of the `summary` function applied to the object created by `lmer` also produces values of AIC and BIC but the definitions used in PROC MIXED and in `lmer` are different. In `lmer` the definitions are such that “smaller is better”. In PROC MIXED the definitions are such that “bigger is better”.

When models are fit by REML, the values of AIC, SBC (or BIC) and the log-likelihood can only be compared between models with exactly the same fixed-effects structure. When models are fit by maximum likelihood these criteria can be compared between any models fit to the same data. That is, these quality-of-fit criteria can be used to evaluate different fixed-effects specifications or different random-effects specifications or different specifications of both fixed effects and random effects. The greater flexibility of model

comparisons when using maximum likelihood is the reason that this is the default criterion in `lmer`.

We encourage developing and testing the model using likelihood ratio tests or the AIC and BIC criteria. Once a form for both the random effects and the fixed effects has been determined, the model can be refit with `REML = TRUE` if the restricted estimates of the variance components are desired.

4 Data manipulation

Both `PROC MIXED` and `lmer` work with data in a tabular form with one row per observation. There are, however, important differences in the internal representations of variables in the data.

In SAS a qualitative factor can be stored either as numerical values or alphanumeric labels. When a factor stored as numerical values is used in `PROC MIXED` it is listed in the `class` statement to indicate that it is a factor. In S this information is stored with the data itself by converting the variable to a factor when it is first stored. If the factor represents an ordered set of levels, it should be converted to an `ordered` factor.

For example the SAS code

```
data animal;
  input trait animal y;
  datalines;
1 1 6
1 2 8
1 3 7
2 1 9
2 2 5
2 3 .
;
```

would require that the `trait` and `animal` variables be specified in a `class` statement in any model that is fit.

In S these data could be read from a file, say `animal.dat`, and converted to factors by

```
animal <- read.table("animal.dat", header = TRUE)
animal$trait <- as.factor(animal$trait)
animal$animal <- as.factor(animal$animal)
```

In general it is a good idea to check the types of variables in a data frame before working with it. One way of doing this is to apply the function `data.class` to each variable in turn using the `sapply` function.

```
> sapply(Animal, data.class)
      Sire      Dam AvgDailyGain
      "factor"    "factor"    "numeric"
> str(Animal)
'data.frame':      20 obs. of  3 variables:
 $ Sire      : Factor w/ 5 levels "1","2","3","4",...: 1 1 1 1 2 2 2 2 3 3 ...
 $ Dam       : Factor w/ 2 levels "1","2": 1 1 2 2 1 1 2 2 1 1 ...
 $ AvgDailyGain: num  2.24 1.85 2.05 2.41 1.99 1.93 2.72 2.32 2.33 2.68 ...
- attr(*, "ginfo")=List of 7
 ..$ formula      :Class 'formula' length 3 AvgDailyGain ~ 1 | Sire/Dam
 .. ..- attr(*, ".Environment")=length 0 <environment>
 ..$ order.groups:List of 2
 .. ..$ Sire: logi TRUE
 .. ..$ Dam : logi TRUE
 ..$ FUN          :function (x)
 ..$ outer        : NULL
 ..$ inner        : NULL
 ..$ labels       :List of 1
 .. ..$ AvgDailyGain: chr "Average Daily Weight Gain"
 ..$ units        : list()
```

To make specification of models in `lmer` easier and to make graphic presentations more informative, we recommend converting from a `data.frame` object to a `groupedData` object. This class of objects contains a formula specifying the response, the primary covariate (if there is one) and the grouping factor or factors. The data sets from Littell et al. (1996) have been converted to `groupedData` objects in this directory.

4.1 Unique levels of factors

Designs with nested grouping factors are indicated differently in the two languages. An example of such an experimental design is the semiconductor experiment described in section 2.2 of Littell et al. (1996) where twelve wafers are assigned to four experimental treatments with three wafers per treatment. The levels for the wafer factor are 1, 2, and 3 but the wafer factor is only meaningful within the same level of the treatment factor, **et**. There is nothing

associating wafer 1 of the third treatment group with wafer 1 of the first treatment group.

In SAS this nesting of factors is denoted by `wafer(et)`. In S the nesting is written with `ET/Wafer` and read “wafer within ET”. If both levels of nested factors are to be associated with random effects then this is all you need to know. You would use an expression with a `"/"` in the grouping factor part of the formula for the `groupedData` object. Then the random effects could be specified as

```
random = list( ET = ~ 1, Wafer = ~ 1 )
```

or, equivalently

```
random = ~ 1 | ET/Wafer
```

In this case, however, there would not usually be any random effects associated with the “experimental treatment” or ET factor. The only random effects are at the `Wafer` level. It is necessary to create a factor that will have unique levels for each `Wafer` within each level of ET. One way to do this is to assign

```
> Semiconductor$Grp <- with(Semiconductor, ET:Wafer)
```

after which we could specify a random effects term of `(1 | Grp)`.

4.2 General approach

As a general approach to importing data into S for mixed-effects analysis you should:

- Create a `data.frame` with one row per observation and one column per variable.
- Use `ordered` or `as.ordered` to explicitly convert any ordered factors to class `ordered`.
- Use `ordered` or `as.ordered` to explicitly convert any ordered factors to class `ordered`.
- If necessary, use `getGroups` to create a factor with unique levels from inner nested factors.
- Specify the formula for the response, the primary covariate and the grouping structure to create a `groupedData` object from the data frame. Labels and units for the response and the primary covariate can also be specified at this time as can `outer` and `inner` factor expressions.

- Plot the data. Plot it several ways. The use of trellis graphics is closely integrated with the `nlme` library. The trellis plots can provide invaluable insight into the structure of the data. Use them.

5 Contrasts

When comparing estimates produced by `SAS PROC MIXED` and by `lmer` one must be careful to consider the contrasts that are used to define the effects of factors. In `SAS` a model with an intercept and a qualitative factor is defined in terms of the intercept and the indicator variables for all but the last level of the factor. The default behaviour in `S` is to use the Helmert contrasts for the factor. On a balanced factor these provide a set of orthogonal contrasts. In `R` the default is the “treatment” contrasts which are almost the same as the `SAS` parameterization except that they drop the indicator of the first level, not the last level.

When in doubt, check which contrasts are being used with the `contrasts` function.

To make comparisons easier, you may find it worthwhile to declare
> options(contrasts = c(factor = "contr.SAS", ordered = "contr.poly"))

at the beginning of your session.

References

Nan M. Laird and James H. Ware. Random-effects models for longitudinal data. *Biometrics*, 38:963–974, 1982.

Ramon C. Littell, George A. Milliken, Walter W. Stroup, and Russell D. Wolfinger. *SAS System for Mixed Models*. SAS Institute, Inc., 1996.

A AvgDailyGain

```
> print(xyplot(adg ~ Treatment | Block, AvgDailyGain, type = c("g",  
+      "p", "r"), xlab = "Treatment (amount of feed additive)",  
+      ylab = "Average daily weight gain (lb.)", aspect = "xy",  
+      index.cond = function(x, y) coef(lm(y ~ x))[1]))
```

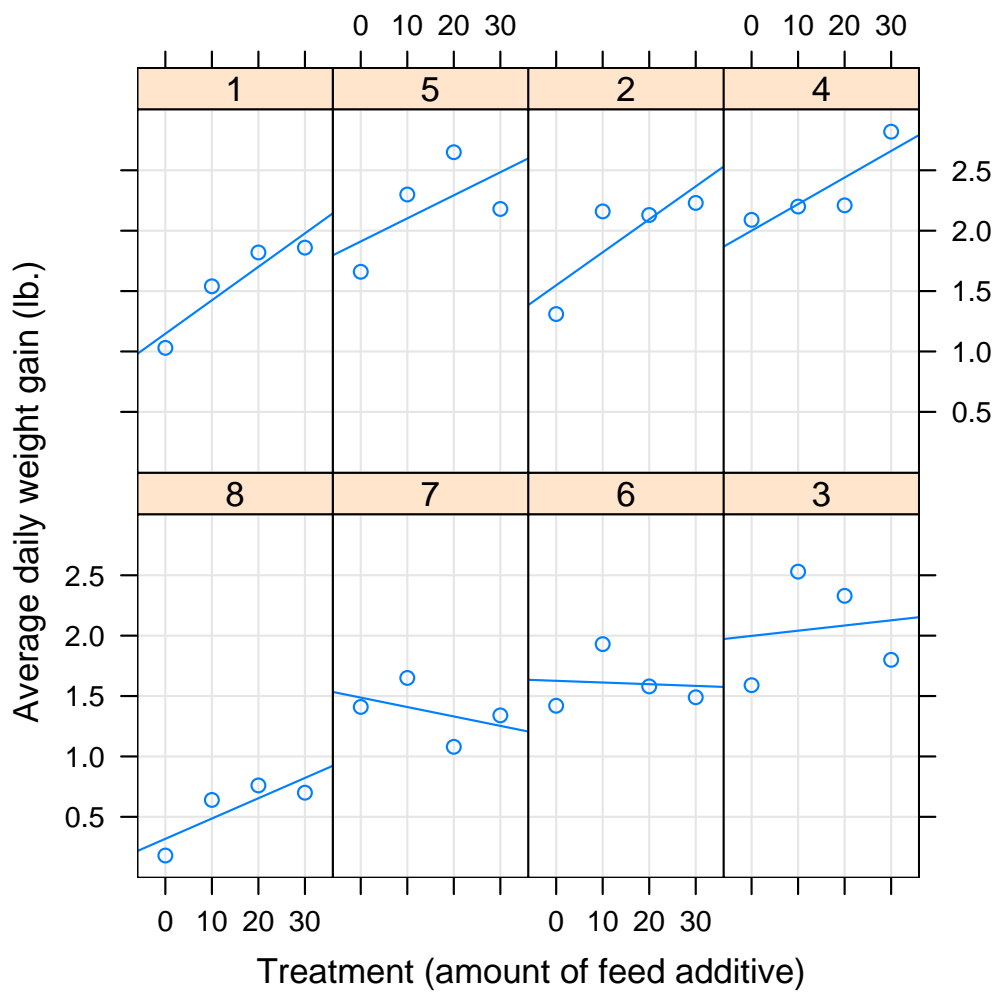


Figure 1: Average daily weight gain

```

> (fmlAdg <- lmer(adg ~ (Treatment - 1) * InitWt + (1 | Block),
+   AvgDailyGain))
Linear mixed-effects model fit by REML
Formula: adg ~ (Treatment - 1) * InitWt + (1 | Block)
Data: AvgDailyGain
   AIC   BIC logLik MLdeviance REMLdeviance
83.33 96.52 -32.66    10.10      65.33
Random effects:
Groups   Name             Variance Std.Dev.
Block    (Intercept)  0.25930   0.50922
Residual                    0.04943   0.22233
number of obs: 32, groups: Block, 8

Fixed effects:
              Estimate Std. Error t value
Treatment0      0.439128   0.711092   0.6175
Treatment10     1.426113   0.637549   2.2369
Treatment20     0.479621   0.548889   0.8738
Treatment30     0.200115   0.775204   0.2581
InitWt          0.004448   0.002082   2.1368
Treatment0:InitWt -0.002154   0.002786  -0.7732
Treatment10:InitWt -0.003365   0.002515  -1.3381
Treatment20:InitWt -0.001082   0.002488  -0.4351

Correlation of Fixed Effects:
              Trtmn0 Trtm10 Trtm20 Trtm30 InitWt Tr0:IW T10:IW
Treatment10   0.039
Treatment20   0.080  0.334
Treatment30   0.011  0.097  0.043
InitWt        0.050 -0.032  0.035 -0.967
Trtmnt0:InW  -0.640  0.046 -0.024  0.754 -0.780
Trtmnt10:IW  -0.021 -0.535 -0.178  0.781 -0.808  0.617
Trtmnt20:IW  -0.040 -0.106 -0.512  0.828 -0.856  0.666  0.775
> anova(fmlAdg)
Analysis of Variance Table
              Df Sum Sq Mean Sq
Treatment      4  5.7250   1.4313
InitWt         1  0.5495   0.5495
Treatment:InitWt 3  0.1381   0.0460
> (fm2Adg <- lmer(adg ~ InitWt + Treatment + (1 | Block), AvgDailyGain))

```



```

Linear mixed-effects model fit by REML
Formula: adg ~ InitWt + Treatment + (1 | Block)
  Data: AvgDailyGain
    AIC   BIC logLik MLdeviance REMLdeviance
48.34 57.13 -18.17    13.62      36.34
Random effects:
Groups   Name             Variance Std.Dev.
Block    (Intercept)  0.24084  0.49076
Residual                    0.05008  0.22379
number of obs: 32, groups: Block, 8

Fixed effects:
              Estimate Std. Error t value
(Intercept)  0.8011075  0.3556610   2.252
InitWt        0.0027797  0.0008334   3.336
Treatment0   -0.5520737  0.1148132  -4.808
Treatment10  -0.0685662  0.1189691  -0.576
Treatment20  -0.0881292  0.1162879  -0.758

Correlation of Fixed Effects:
              (Intr) InitWt Trtmn0 Trtm10
InitWt        -0.844
Treatment0     0.036 -0.224
Treatment10    0.139 -0.340  0.534
Treatment20    0.079 -0.272  0.530  0.545
> anova(fm2Adg)
Analysis of Variance Table
              Df  Sum Sq Mean Sq
InitWt         1  0.51456  0.51456
Treatment      3  1.52670  0.50890
> (fm3Adg <- lmer(adg ~ InitWt + Treatment - 1 + (1 | Block),
+   AvgDailyGain))
Linear mixed-effects model fit by REML
Formula: adg ~ InitWt + Treatment - 1 + (1 | Block)
  Data: AvgDailyGain
    AIC   BIC logLik MLdeviance REMLdeviance
48.34 57.13 -18.17    13.62      36.34
Random effects:
Groups   Name             Variance Std.Dev.
Block    (Intercept)  0.24084  0.49076

```

```

Residual          0.05008  0.22379
number of obs: 32, groups: Block, 8

```

Fixed effects:

| | Estimate | Std. Error | t value |
|-------------|-----------|------------|---------|
| InitWt | 0.0027797 | 0.0008334 | 3.336 |
| Treatment0 | 0.2490338 | 0.3776318 | 0.659 |
| Treatment10 | 0.7325413 | 0.3903798 | 1.876 |
| Treatment20 | 0.7129784 | 0.3827685 | 1.863 |
| Treatment30 | 0.8011075 | 0.3556610 | 2.252 |

Correlation of Fixed Effects:

| | InitWt | Trtmn0 | Trtml0 | Trtm20 |
|-------------|--------|--------|--------|--------|
| Treatment0 | -0.863 | | | |
| Treatment10 | -0.873 | 0.957 | | |
| Treatment20 | -0.867 | 0.957 | 0.958 | |
| Treatment30 | -0.844 | 0.953 | 0.953 | 0.953 |

B BIB

```

> print(xyplot(y ~ x | Block, BIB, groups = Treatment, type = c("g",
+      "p"), aspect = "xy", auto.key = list(points = TRUE, space = "right",
+      lines = FALSE)))

```

```

> (fm1BIB <- lmer(y ~ Treatment * x + (1 | Block), BIB))

```

Linear mixed-effects model fit by REML

Formula: y ~ Treatment * x + (1 | Block)

Data: BIB

| AIC | BIC | logLik | MLdeviance | REMLdeviance |
|-------|-------|--------|------------|--------------|
| 122.9 | 133.5 | -52.45 | 93.5 | 104.9 |

Random effects:

| Groups | Name | Variance | Std.Dev. |
|----------|-------------|----------|----------|
| Block | (Intercept) | 18.2499 | 4.2720 |
| Residual | | 1.2004 | 1.0956 |

number of obs: 24, groups: Block, 8

Fixed effects:

| | Estimate | Std. Error | t value |
|-------------|----------|------------|---------|
| (Intercept) | 22.36784 | 3.10182 | 7.211 |
| Treatment1 | 4.42949 | 3.36504 | 1.316 |

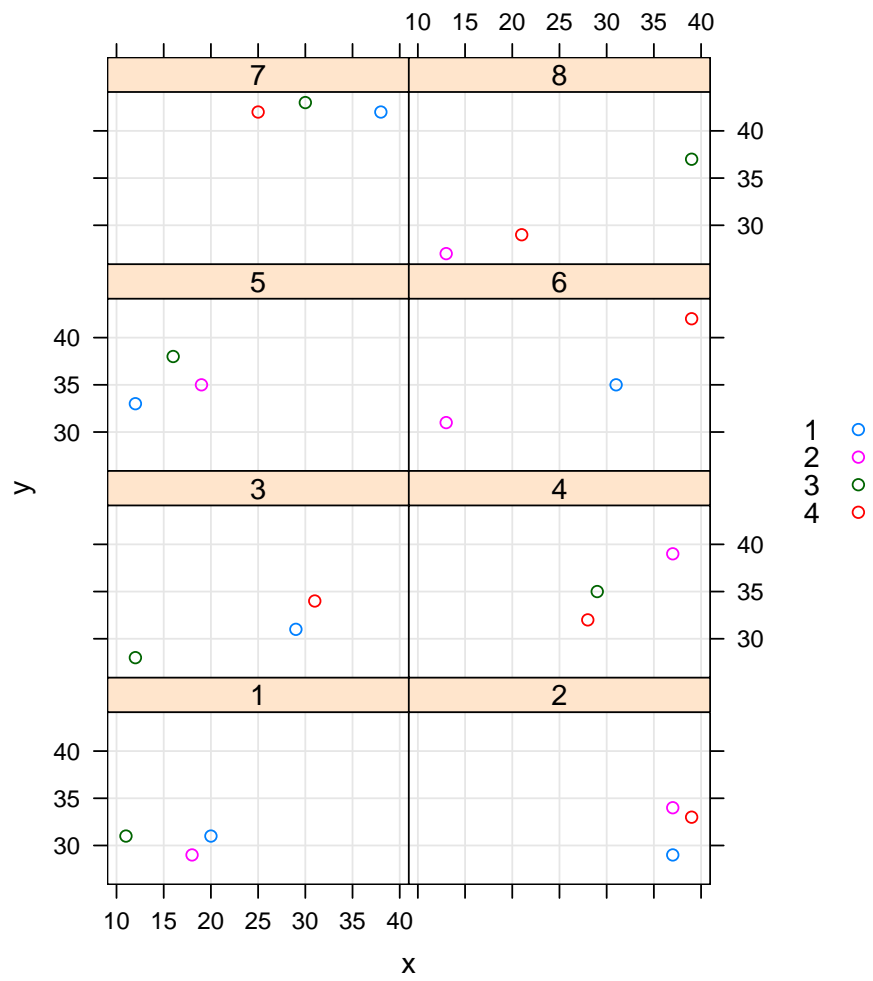


Figure 2: Balanced incomplete block design

| | | | |
|--------------|----------|---------|--------|
| Treatment2 | -0.43737 | 2.93320 | -0.149 |
| Treatment3 | 6.27864 | 3.28203 | 1.913 |
| x | 0.44255 | 0.08706 | 5.083 |
| Treatment1:x | -0.22377 | 0.10608 | -2.109 |
| Treatment2:x | 0.05338 | 0.09714 | 0.550 |
| Treatment3:x | -0.17918 | 0.11571 | -1.549 |

Correlation of Fixed Effects:

| | (Intr) | Trtmn1 | Trtmn2 | Trtmn3 | x | Trtm1: | Trtm2: |
|-------------|--------|--------|--------|--------|--------|--------|--------|
| Treatment1 | -0.728 | | | | | | |
| Treatment2 | -0.778 | 0.797 | | | | | |
| Treatment3 | -0.796 | 0.827 | 0.826 | | | | |
| x | -0.859 | 0.797 | 0.865 | 0.886 | | | |
| Treatmnt1:x | 0.709 | -0.979 | -0.774 | -0.797 | -0.799 | | |
| Treatmnt2:x | 0.722 | -0.731 | -0.965 | -0.763 | -0.829 | 0.729 | |
| Treatmnt3:x | 0.769 | -0.789 | -0.790 | -0.976 | -0.879 | 0.777 | 0.748 |

> anova(fm1BIB)

Analysis of Variance Table

| | Df | Sum Sq | Mean Sq |
|-------------|----|---------|---------|
| Treatment | 3 | 23.447 | 7.816 |
| x | 1 | 136.809 | 136.809 |
| Treatment:x | 3 | 18.427 | 6.142 |

> (fm2BIB <- lmer(y ~ Treatment + x:Grp + (1 | Block), BIB))

Linear mixed-effects model fit by REML

Formula: y ~ Treatment + x:Grp + (1 | Block)

Data: BIB

| AIC | BIC | logLik | MLdeviance | REMLdeviance |
|-------|-------|--------|------------|--------------|
| 113.2 | 121.4 | -49.59 | 94.09 | 99.18 |

Random effects:

| Groups | Name | Variance | Std.Dev. |
|----------|-------------|----------|----------|
| Block | (Intercept) | 18.5214 | 4.3036 |
| Residual | | 1.0380 | 1.0188 |

number of obs: 24, groups: Block, 8

Fixed effects:

| | Estimate | Std. Error | t value |
|-------------|----------|------------|---------|
| (Intercept) | 20.94523 | 2.06223 | 10.157 |
| Treatment1 | 5.34139 | 1.97584 | 2.703 |
| Treatment2 | 1.13555 | 0.71404 | 1.590 |
| Treatment3 | 8.18098 | 1.77022 | 4.621 |

```

x:Grp13      0.23952      0.04297      5.575
x:Grp24      0.48923      0.04412     11.087

Correlation of Fixed Effects:
      (Intr) Trtmn1 Trtmn2 Trtmn3 x:Gr13
Treatment1 -0.501
Treatment2 -0.431  0.559
Treatment3 -0.527  0.942  0.581
x:Grp13     0.027 -0.663 -0.165 -0.605
x:Grp24    -0.639  0.651  0.452  0.688  0.042
> anova(fm2BIB)
Analysis of Variance Table
      Df  Sum Sq Mean Sq
Treatment  3  23.424    7.808
x:Grp      2 154.733   77.366

```

C Bond

```

> (fm1Bond <- lmer(pressure ~ Metal + (1 | Ingot), Bond))
Linear mixed-effects model fit by REML
Formula: pressure ~ Metal + (1 | Ingot)
Data: Bond
      AIC      BIC logLik MLdeviance REMLdeviance
115.8 120.0  -53.9      115.7       107.8

Random effects:
Groups   Name      Variance Std.Dev.
Ingot    (Intercept) 11.448    3.3835
Residual                10.372    3.2205
number of obs: 21, groups: Ingot, 7

```

```

Fixed effects:
      Estimate Std. Error t value
(Intercept)  71.1000      1.7655  40.27
Metalc       -0.9143      1.7214  -0.53
Metali        4.8000      1.7214   2.79

```

```

Correlation of Fixed Effects:
      (Intr) Metalc
Metalc -0.488
Metali -0.488  0.500

```

```
> anova(fmlBond)
Analysis of Variance Table

      Df Sum Sq Mean Sq
Metal  2 131.90   65.95
```

D Cultivation

```
> str(Cultivation)
'data.frame':      24 obs. of  4 variables:
 $ Block: Factor w/ 4 levels "1","2","3","4": 1 1 1 1 1 1 2 2 2 2 ...
 $ Cult : Factor w/ 2 levels "a","b": 1 1 1 2 2 2 1 1 1 2 ...
 $ Inoc : Factor w/ 3 levels "con","dea","liv": 1 2 3 1 2 3 1 2 3 1 ...
 $ drywt: num  27.4 29.7 34.5 29.4 32.5 34.4 28.9 28.7 33.4 28.7 ...
- attr(*, "ginfo")=List of 7
 ..$ formula      :Class 'formula' length 3 drywt ~ 1 | Block/Cult
 .. ..- attr(*, ".Environment")=length 7 <environment>
 ..$ order.groups:List of 2
 .. ..$ Block: logi TRUE
 .. ..$ Cult : logi TRUE
 ..$ FUN          :function (x)
 ..$ outer        : NULL
 ..$ inner        :List of 1
 .. ..$ Cult:Class 'formula' length 2 ~Inoc
 .. ..- attr(*, ".Environment")=length 7 <environment>
 ..$ labels       :List of 1
 .. ..$ drywt: chr "Yield"
 ..$ units        : list()
> xtabs(~Block + Cult, Cultivation)
      Cult
Block a b
  1 3 3
  2 3 3
  3 3 3
  4 3 3
> (fmlCult <- lmer(drywt ~ Inoc * Cult + (1 | Block) + (1 |
+      Cult), Cultivation))
Linear mixed-effects model fit by REML
Formula: drywt ~ Inoc * Cult + (1 | Block) + (1 | Cult)
Data: Cultivation
AIC    BIC logLik MLdeviance REMLdeviance
```

```

84.49 93.91 -34.24      76.7      68.49
Random effects:
  Groups   Name      Variance Std.Dev.
Block    (Intercept) 1.2073   1.0988
Cult      (Intercept) 1.0634   1.0312
Residual                1.1963   1.0938
number of obs: 24, groups: Block, 4; Cult, 2

```

```

Fixed effects:
              Estimate Std. Error t value
(Intercept)   33.5250     1.2901  25.987
Inoccon       -5.5000     0.7734  -7.111
Inocdea       -2.8750     0.7734  -3.717
Culta         -0.3750     1.6508  -0.227
Inoccon:Culta  0.2500     1.0938   0.229
Inocdea:Culta -1.0250     1.0938  -0.937

```

```

Correlation of Fixed Effects:
              (Intr) Inoccn Inocde Culta  Incc:C
Inoccon      -0.300
Inocdea      -0.300  0.500
Culta        -0.640  0.234  0.234
Inoccon:Clt  0.212 -0.707 -0.354 -0.331
Inocdea:Clt  0.212 -0.354 -0.707 -0.331  0.500
> anova(fm1Cult)

```

Analysis of Variance Table

```

              Df Sum Sq Mean Sq
Inoc          2 118.176   59.088
Cult          1   0.206    0.206
Inoc:Cult     2   1.826    0.913

```

```

> (fm2Cult <- lmer(drywt ~ Inoc + Cult + (1 | Block) + (1 |
+      Cult), Cultivation))

```

Linear mixed-effects model fit by REML

```

Formula: drywt ~ Inoc + Cult + (1 | Block) + (1 | Cult)

```

```

Data: Cultivation

```

```

      AIC      BIC logLik MLdeviance REMLdeviance
85.75 92.82 -36.88      78.65      73.75

```

Random effects:

```

  Groups   Name      Variance Std.Dev.
Block    (Intercept) 1.2128   1.1013

```

| | | | |
|----------|-------------|--------|--------|
| Cult | (Intercept) | 1.0338 | 1.0167 |
| Residual | | 1.1630 | 1.0784 |

number of obs: 24, groups: Block, 4; Cult, 2

Fixed effects:

| | Estimate | Std. Error | t value |
|-------------|----------|------------|---------|
| (Intercept) | 33.6542 | 1.2373 | 27.201 |
| Inoccon | -5.3750 | 0.5392 | -9.968 |
| Inocdea | -3.3875 | 0.5392 | -6.282 |
| Culta | -0.6333 | 1.5038 | -0.421 |

Correlation of Fixed Effects:

| | (Intr) | Inoccn | Inocde |
|---------|--------|--------|--------|
| Inoccon | -0.218 | | |
| Inocdea | -0.218 | 0.500 | |
| Culta | -0.608 | 0.000 | 0.000 |

> anova(fm2Cult)

Analysis of Variance Table

| | Df | Sum Sq | Mean Sq |
|------|----|---------|---------|
| Inoc | 2 | 118.176 | 59.088 |
| Cult | 1 | 0.206 | 0.206 |

> (fm3Cult <- lmer(drywt ~ Inoc + (1 | Block) + (1 | Cult),
+ Cultivation))

Linear mixed-effects model fit by REML

Formula: drywt ~ Inoc + (1 | Block) + (1 | Cult)

Data: Cultivation

| AIC | BIC | logLik | MLdeviance | REMLdeviance |
|-------|-------|--------|------------|--------------|
| 85.68 | 91.57 | -37.84 | 77.32 | 75.68 |

Random effects:

| Groups | Name | Variance | Std.Dev. |
|----------|-------------|----------|----------|
| Block | (Intercept) | 1.21283 | 1.10129 |
| Cult | (Intercept) | 0.10364 | 0.32193 |
| Residual | | 1.16299 | 1.07842 |

number of obs: 24, groups: Block, 4; Cult, 2

Fixed effects:

| | Estimate | Std. Error | t value |
|-------------|----------|------------|---------|
| (Intercept) | 33.3375 | 0.7074 | 47.13 |
| Inoccon | -5.3750 | 0.5392 | -9.97 |
| Inocdea | -3.3875 | 0.5392 | -6.28 |

Correlation of Fixed Effects:

```

      (Intr) Inoccn
Inoccon -0.381
Inocdea -0.381  0.500

```

```
> anova(fm3Cult)
```

Analysis of Variance Table

```

      Df  Sum Sq Mean Sq
Inoc   2 118.176  59.088

```

E Demand

```
> (fmlDemand <- lmer(log(d) ~ log(y) + log(rd) + log(rt) +
+   log(rs) + (1 | State) + (1 | Year), Demand))
```

Linear mixed-effects model fit by REML

Formula: log(d) ~ log(y) + log(rd) + log(rt) + log(rs) + (1 | State) +

Data: Demand

```

      AIC      BIC logLik MLdeviance REMLdeviance
-226.2 -209.8  120.1    -260.5      -240.2

```

Random effects:

```

Groups   Name             Variance Std.Dev.
Year     (Intercept) 0.00026466 0.016268
State    (Intercept) 0.02950854 0.171781
Residual                    0.00111697 0.033421

```

number of obs: 77, groups: Year, 11; State, 7

Fixed effects:

```

      Estimate Std. Error t value
(Intercept) -1.28377    0.72344  -1.775
log(y)       1.06977    0.10393  10.294
log(rd)     -0.29532    0.05246  -5.629
log(rt)      0.03988    0.02789   1.430
log(rs)     -0.32673    0.11438  -2.856

```

Correlation of Fixed Effects:

```

      (Intr) log(y) lg(rd) lg(rt)
log(y)  -0.976
log(rd)  0.383 -0.227
log(rt)  0.077 -0.062 -0.337
log(rs)  0.444 -0.600 -0.270 -0.323

```

F HR

```
> (fm1HR <- lmer(HR ~ Time * Drug + baseHR + (Time | Patient),
+               HR))
```

Linear mixed-effects model fit by REML

Formula: HR ~ Time * Drug + baseHR + (Time | Patient)

Data: HR

| AIC | BIC | logLik | MLdeviance | REMLdeviance |
|-------|-------|--------|------------|--------------|
| 787.6 | 815.5 | -383.8 | 788.1 | 767.6 |

Random effects:

| Groups | Name | Variance | Std.Dev. | Corr |
|----------|-------------|----------|----------|--------|
| Patient | (Intercept) | 60.633 | 7.7867 | |
| | Time | 37.784 | 6.1469 | -0.563 |
| Residual | | 24.361 | 4.9357 | |

number of obs: 120, groups: Patient, 24

Fixed effects:

| | Estimate | Std. Error | t value |
|-------------|----------|------------|---------|
| (Intercept) | 33.9776 | 10.2830 | 3.304 |
| Time | -3.1970 | 3.0849 | -1.036 |
| Druga | 3.5992 | 4.2314 | 0.851 |
| Drugb | 7.0912 | 4.2094 | 1.685 |
| baseHR | 0.5434 | 0.1161 | 4.679 |
| Time:Druga | -7.5013 | 4.3627 | -1.719 |
| Time:Drugb | -3.9894 | 4.3627 | -0.914 |

Correlation of Fixed Effects:

| | (Intr) | Time | Druga | Drugb | baseHR | Tim:Drg |
|------------|--------|--------|--------|--------|--------|---------|
| Time | -0.162 | | | | | |
| Druga | -0.308 | 0.394 | | | | |
| Drugb | -0.244 | 0.396 | 0.501 | | | |
| baseHR | -0.957 | 0.000 | 0.110 | 0.041 | | |
| Time:Druga | 0.115 | -0.707 | -0.557 | -0.280 | 0.000 | |
| Time:Drugb | 0.115 | -0.707 | -0.278 | -0.560 | 0.000 | 0.500 |

```
> anova(fm1HR)
```

Analysis of Variance Table

| | Df | Sum Sq | Mean Sq |
|-----------|----|--------|---------|
| Time | 1 | 379.23 | 379.23 |
| Drug | 2 | 92.88 | 46.44 |
| baseHR | 1 | 533.27 | 533.27 |
| Time:Drug | 2 | 72.12 | 36.06 |

```
> (fm3HR <- lmer(HR ~ Time + Drug + baseHR + (Time | Patient),
+ HR))
```

Linear mixed-effects model fit by REML

Formula: HR ~ Time + Drug + baseHR + (Time | Patient)

Data: HR

| AIC | BIC | logLik | MLdeviance | REMLdeviance |
|-------|-------|--------|------------|--------------|
| 795.8 | 818.1 | -389.9 | 791.2 | 779.8 |

Random effects:

| Groups | Name | Variance | Std.Dev. | Corr |
|----------|-------------|----------|----------|--------|
| Patient | (Intercept) | 61.560 | 7.8460 | |
| | Time | 40.963 | 6.4003 | -0.570 |
| Residual | | 24.361 | 4.9357 | |

number of obs: 120, groups: Patient, 24

Fixed effects:

| | Estimate | Std. Error | t value |
|-------------|----------|------------|---------|
| (Intercept) | 36.0464 | 10.1945 | 3.536 |
| Time | -7.0273 | 1.8179 | -3.866 |
| Druga | -0.4524 | 3.5146 | -0.129 |
| Drugb | 4.9365 | 3.4881 | 1.415 |
| baseHR | 0.5434 | 0.1161 | 4.679 |

Correlation of Fixed Effects:

| | (Intr) Time | Druga | Drugb |
|--------|-------------|-------|-------|
| Time | -0.096 | | |
| Druga | -0.297 | 0.000 | |
| Drugb | -0.219 | 0.000 | 0.502 |
| baseHR | -0.966 | 0.000 | 0.132 |

```
> anova(fm3HR)
```

Analysis of Variance Table

| | Df | Sum Sq | Mean Sq |
|--------|----|--------|---------|
| Time | 1 | 364.03 | 364.03 |
| Drug | 2 | 92.88 | 46.44 |
| baseHR | 1 | 533.27 | 533.27 |

```
> (fm4HR <- lmer(HR ~ Time + baseHR + (Time | Patient), HR))
```

Linear mixed-effects model fit by REML

Formula: HR ~ Time + baseHR + (Time | Patient)

Data: HR

| AIC | BIC | logLik | MLdeviance | REMLdeviance |
|-------|-------|--------|------------|--------------|
| 803.1 | 819.9 | -395.6 | 794.3 | 791.1 |

Random effects:

| Groups | Name | Variance | Std.Dev. | Corr |
|----------|-------------|----------|----------|--------|
| Patient | (Intercept) | 63.026 | 7.9389 | |
| | Time | 40.963 | 6.4002 | -0.553 |
| Residual | | 24.361 | 4.9357 | |

number of obs: 120, groups: Patient, 24

Fixed effects:

| | Estimate | Std. Error | t value |
|-------------|----------|------------|---------|
| (Intercept) | 36.9314 | 9.9014 | 3.730 |
| Time | -7.0273 | 1.8179 | -3.866 |
| baseHR | 0.5508 | 0.1175 | 4.686 |

Correlation of Fixed Effects:

| | (Intr) | Time |
|--------|--------|-------|
| Time | -0.098 | |
| baseHR | -0.984 | 0.000 |

> anova(fm4HR)

Analysis of Variance Table

| | Df | Sum Sq | Mean Sq |
|--------|----|--------|---------|
| Time | 1 | 364.03 | 364.03 |
| baseHR | 1 | 534.87 | 534.87 |

G Mississippi

```
> (fmlMiss <- lmer(y ~ 1 + (1 | influent), Mississippi))
```

Linear mixed-effects model fit by REML

Formula: y ~ 1 + (1 | influent)

Data: Mississippi

| AIC | BIC | logLik | MLdeviance | REMLdeviance |
|-------|-------|--------|------------|--------------|
| 256.4 | 259.6 | -126.2 | 256.6 | 252.4 |

Random effects:

| Groups | Name | Variance | Std.Dev. |
|----------|-------------|----------|----------|
| influent | (Intercept) | 63.324 | 7.9576 |
| Residual | | 42.658 | 6.5313 |

number of obs: 37, groups: influent, 6

Fixed effects:

| | Estimate | Std. Error | t value |
|-------------|----------|------------|---------|
| (Intercept) | 21.223 | 3.429 | 6.189 |

```

> (fmlMLMiss <- lmer(y ~ 1 + (1 | influent), Mississippi, method = "ML"))
Linear mixed-effects model fit by maximum likelihood
Formula: y ~ 1 + (1 | influent)
Data: Mississippi
AIC      BIC logLik MLdeviance REMLdeviance
260.6 263.8 -128.3      256.6      252.4
Random effects:
Groups      Name      Variance Std.Dev.
influente (Intercept) 51.255    7.1592
Residual                42.697    6.5343
number of obs: 37, groups: influente, 6

Fixed effects:
              Estimate Std. Error t value
(Intercept)   21.217      3.122    6.796
> ranef(fmlMLMiss)
An object of class "ranef.lmer"
[[1]]
      (Intercept)
1      0.3097833
2     -6.5772271
3     -3.7862742
4      2.8826708
5     -5.8435201
6     13.0145672
> ranef(fmlMiss)
An object of class "ranef.lmer"
[[1]]
      (Intercept)
1      0.309286
2     -6.719335
3     -3.897948
4      2.946106
5     -6.012988
6     13.374879
> VarCorr(fmlMiss)
$influente
1 x 1 Matrix of class "dpoMatrix"
      (Intercept)
(Intercept)   63.32364

attr(,"sc")
[1] 6.531315

```

```
> (fm2Miss <- lmer(y ~ Type + (1 | influent), Mississippi))
```

Linear mixed-effects model fit by REML

Formula: y ~ Type + (1 | influent)

Data: Mississippi

| | AIC | BIC | logLik | MLdeviance | REMLdeviance |
|--|-------|-------|--------|------------|--------------|
| | 242.5 | 249.0 | -117.3 | 247.5 | 234.5 |

Random effects:

| Groups | Name | Variance | Std.Dev. |
|----------|-------------|----------|----------|
| influ | (Intercept) | 14.970 | 3.8691 |
| Residual | | 42.514 | 6.5202 |

number of obs: 37, groups: influent, 6

Fixed effects:

| | Estimate | Std. Error | t value |
|-------------|----------|------------|---------|
| (Intercept) | 36.400 | 4.845 | 7.513 |
| Type1 | -20.800 | 5.934 | -3.505 |
| Type2 | -16.462 | 5.517 | -2.984 |

Correlation of Fixed Effects:

| | (Intr) | Type1 |
|-------|--------|-------|
| Type1 | -0.816 | |
| Type2 | -0.878 | 0.717 |

```
> anova(fm2Miss)
```

Analysis of Variance Table

| | Df | Sum Sq | Mean Sq |
|------|----|--------|---------|
| Type | 2 | 541.76 | 270.88 |

H Multilocation

```
> str(Multilocation)
```

'data.frame': 108 obs. of 7 variables:

\$ obs : num 3 4 6 7 9 10 12 16 19 20 ...

\$ Location: Factor w/ 9 levels "A","B","C","D",...: 1 1 1 1 1 1 1 1 1 1 ...

\$ Block : Factor w/ 3 levels "1","2","3": 1 1 1 1 2 2 2 2 3 3 ...

\$ Trt : Factor w/ 4 levels "1","2","3","4": 3 4 2 1 2 1 3 4 1 2 ...

\$ Adj : num 3.16 3.12 3.16 3.25 2.71 ...

\$ Fe : num 7.10 6.68 6.83 6.53 8.25 ...

\$ Grp : Factor w/ 27 levels "A/1","A/2","A/3",...: 1 1 1 1 2 2 2 2 3 3 ...

- attr(*, "ginfo")=List of 7

..\$ formula :Class 'formula' length 3 Adj ~ 1 | Location/Block

```

.. .. - attr(*, ".Environment")=length 17 <environment>
..$ order.groups:List of 2
.. ..$ Location: logi TRUE
.. ..$ Block : logi TRUE
..$ FUN :function (x)
..$ outer : NULL
..$ inner :List of 1
.. ..$ Block:Class 'formula' length 2 ~Trt
.. .. - attr(*, ".Environment")=length 17 <environment>
..$ labels :List of 1
.. ..$ Adj: chr "Adjusted yield"
..$ units : list()
> Multilocation$Grp <- with(Multilocation, Block:Location)
> (fmlMult <- lmer(Adj ~ Location * Trt + (1 | Grp), Multilocation))
Linear mixed-effects model fit by REML
Formula: Adj ~ Location * Trt + (1 | Grp)
Data: Multilocation
AIC BIC logLik MLdeviance REMLdeviance
84.65 183.9 -5.323 -87.15 10.65
Random effects:
Groups Name Variance Std.Dev.
Grp (Intercept) 0.0056193 0.074962
Residual 0.0345787 0.185953
number of obs: 108, groups: Grp, 27

Fixed effects:

```

| | Estimate | Std. Error | t value |
|----------------|----------|------------|---------|
| (Intercept) | 2.35923 | 0.11575 | 20.381 |
| LocationA | 0.64930 | 0.16370 | 3.966 |
| LocationB | 0.06643 | 0.16370 | 0.406 |
| LocationC | 0.54533 | 0.16370 | 3.331 |
| LocationD | 0.37413 | 0.16370 | 2.285 |
| LocationE | 0.55000 | 0.16370 | 3.360 |
| LocationF | 0.99810 | 0.16370 | 6.097 |
| LocationG | 0.36057 | 0.16370 | 2.203 |
| LocationH | 1.01403 | 0.16370 | 6.194 |
| Trt1 | 0.22720 | 0.15183 | 1.496 |
| Trt2 | -0.00140 | 0.15183 | -0.009 |
| Trt3 | 0.42323 | 0.15183 | 2.788 |
| LocationA:Trt1 | -0.18853 | 0.21472 | -0.878 |

| | | | |
|----------------|----------|---------|--------|
| LocationB:Trt1 | -0.27523 | 0.21472 | -1.282 |
| LocationC:Trt1 | -0.04000 | 0.21472 | -0.186 |
| LocationD:Trt1 | -0.53513 | 0.21472 | -2.492 |
| LocationE:Trt1 | -0.26297 | 0.21472 | -1.225 |
| LocationF:Trt1 | -0.27153 | 0.21472 | -1.265 |
| LocationG:Trt1 | 0.20323 | 0.21472 | 0.947 |
| LocationH:Trt1 | -0.14953 | 0.21472 | -0.696 |
| LocationA:Trt2 | -0.09347 | 0.21472 | -0.435 |
| LocationB:Trt2 | -0.32273 | 0.21472 | -1.503 |
| LocationC:Trt2 | 0.08960 | 0.21472 | 0.417 |
| LocationD:Trt2 | -0.29693 | 0.21472 | -1.383 |
| LocationE:Trt2 | -0.30693 | 0.21472 | -1.429 |
| LocationF:Trt2 | -0.30993 | 0.21472 | -1.443 |
| LocationG:Trt2 | -0.10860 | 0.21472 | -0.506 |
| LocationH:Trt2 | -0.33060 | 0.21472 | -1.540 |
| LocationA:Trt3 | -0.40247 | 0.21472 | -1.874 |
| LocationB:Trt3 | -0.56550 | 0.21472 | -2.634 |
| LocationC:Trt3 | -0.12247 | 0.21472 | -0.570 |
| LocationD:Trt3 | -0.54840 | 0.21472 | -2.554 |
| LocationE:Trt3 | -0.32863 | 0.21472 | -1.531 |
| LocationF:Trt3 | -0.46257 | 0.21472 | -2.154 |
| LocationG:Trt3 | -0.25297 | 0.21472 | -1.178 |
| LocationH:Trt3 | -0.37203 | 0.21472 | -1.733 |

Correlation of Fixed Effects:

| | (Intr) | LoctnA | LoctnB | LoctnC | LoctnD | LoctnE | LoctnF | LoctnG | LoctnH |
|-------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| LocationA | -0.707 | | | | | | | | |
| LocationB | -0.707 | 0.500 | | | | | | | |
| LocationC | -0.707 | 0.500 | 0.500 | | | | | | |
| LocationD | -0.707 | 0.500 | 0.500 | 0.500 | | | | | |
| LocationE | -0.707 | 0.500 | 0.500 | 0.500 | 0.500 | | | | |
| LocationF | -0.707 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | | | |
| LocationG | -0.707 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | | |
| LocationH | -0.707 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | |
| Trt1 | -0.656 | 0.464 | 0.464 | 0.464 | 0.464 | 0.464 | 0.464 | 0.464 | 0.464 |
| Trt2 | -0.656 | 0.464 | 0.464 | 0.464 | 0.464 | 0.464 | 0.464 | 0.464 | 0.464 |
| Trt3 | -0.656 | 0.464 | 0.464 | 0.464 | 0.464 | 0.464 | 0.464 | 0.464 | 0.464 |
| LoctnA:Trt1 | 0.464 | -0.656 | -0.328 | -0.328 | -0.328 | -0.328 | -0.328 | -0.328 | -0.328 |
| LoctnB:Trt1 | 0.464 | -0.328 | -0.656 | -0.328 | -0.328 | -0.328 | -0.328 | -0.328 | -0.328 |
| LoctnC:Trt1 | 0.464 | -0.328 | -0.328 | -0.656 | -0.328 | -0.328 | -0.328 | -0.328 | -0.328 |

| | | | | | | | | | |
|-------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| LoctnD:Trt1 | 0.464 | -0.328 | -0.328 | -0.328 | -0.656 | -0.328 | -0.328 | -0.328 | -0.328 |
| LoctnE:Trt1 | 0.464 | -0.328 | -0.328 | -0.328 | -0.328 | -0.656 | -0.328 | -0.328 | -0.328 |
| LoctnF:Trt1 | 0.464 | -0.328 | -0.328 | -0.328 | -0.328 | -0.328 | -0.656 | -0.328 | -0.328 |
| LoctnG:Trt1 | 0.464 | -0.328 | -0.328 | -0.328 | -0.328 | -0.328 | -0.328 | -0.656 | -0.328 |
| LoctnH:Trt1 | 0.464 | -0.328 | -0.328 | -0.328 | -0.328 | -0.328 | -0.328 | -0.328 | -0.656 |
| LoctnA:Trt2 | 0.464 | -0.656 | -0.328 | -0.328 | -0.328 | -0.328 | -0.328 | -0.328 | -0.328 |
| LoctnB:Trt2 | 0.464 | -0.328 | -0.656 | -0.328 | -0.328 | -0.328 | -0.328 | -0.328 | -0.328 |
| LoctnC:Trt2 | 0.464 | -0.328 | -0.328 | -0.656 | -0.328 | -0.328 | -0.328 | -0.328 | -0.328 |
| LoctnD:Trt2 | 0.464 | -0.328 | -0.328 | -0.328 | -0.656 | -0.328 | -0.328 | -0.328 | -0.328 |
| LoctnE:Trt2 | 0.464 | -0.328 | -0.328 | -0.328 | -0.328 | -0.656 | -0.328 | -0.328 | -0.328 |
| LoctnF:Trt2 | 0.464 | -0.328 | -0.328 | -0.328 | -0.328 | -0.328 | -0.656 | -0.328 | -0.328 |
| LoctnG:Trt2 | 0.464 | -0.328 | -0.328 | -0.328 | -0.328 | -0.328 | -0.328 | -0.656 | -0.328 |
| LoctnH:Trt2 | 0.464 | -0.328 | -0.328 | -0.328 | -0.328 | -0.328 | -0.328 | -0.328 | -0.656 |
| LoctnA:Trt3 | 0.464 | -0.656 | -0.328 | -0.328 | -0.328 | -0.328 | -0.328 | -0.328 | -0.328 |
| LoctnB:Trt3 | 0.464 | -0.328 | -0.656 | -0.328 | -0.328 | -0.328 | -0.328 | -0.328 | -0.328 |
| LoctnC:Trt3 | 0.464 | -0.328 | -0.328 | -0.656 | -0.328 | -0.328 | -0.328 | -0.328 | -0.328 |
| LoctnD:Trt3 | 0.464 | -0.328 | -0.328 | -0.328 | -0.656 | -0.328 | -0.328 | -0.328 | -0.328 |
| LoctnE:Trt3 | 0.464 | -0.328 | -0.328 | -0.328 | -0.328 | -0.656 | -0.328 | -0.328 | -0.328 |
| LoctnF:Trt3 | 0.464 | -0.328 | -0.328 | -0.328 | -0.328 | -0.328 | -0.656 | -0.328 | -0.328 |
| LoctnG:Trt3 | 0.464 | -0.328 | -0.328 | -0.328 | -0.328 | -0.328 | -0.328 | -0.656 | -0.328 |
| LoctnH:Trt3 | 0.464 | -0.328 | -0.328 | -0.328 | -0.328 | -0.328 | -0.328 | -0.328 | -0.656 |
| | Trt1 | Trt2 | Trt3 | LcA:T1 | LcB:T1 | LcC:T1 | LcD:T1 | LcE:T1 | LcF:T1 |
| LocationA | | | | | | | | | |
| LocationB | | | | | | | | | |
| LocationC | | | | | | | | | |
| LocationD | | | | | | | | | |
| LocationE | | | | | | | | | |
| LocationF | | | | | | | | | |
| LocationG | | | | | | | | | |
| LocationH | | | | | | | | | |
| Trt1 | | | | | | | | | |
| Trt2 | | 0.500 | | | | | | | |
| Trt3 | | 0.500 | 0.500 | | | | | | |
| LoctnA:Trt1 | -0.707 | -0.354 | -0.354 | | | | | | |
| LoctnB:Trt1 | -0.707 | -0.354 | -0.354 | 0.500 | | | | | |
| LoctnC:Trt1 | -0.707 | -0.354 | -0.354 | 0.500 | 0.500 | | | | |
| LoctnD:Trt1 | -0.707 | -0.354 | -0.354 | 0.500 | 0.500 | 0.500 | | | |
| LoctnE:Trt1 | -0.707 | -0.354 | -0.354 | 0.500 | 0.500 | 0.500 | 0.500 | | |
| LoctnF:Trt1 | -0.707 | -0.354 | -0.354 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | |
| LoctnG:Trt1 | -0.707 | -0.354 | -0.354 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 |

| | | | | | | | | | |
|-------------|--------|--------|--------|--------|--------|--------|--------|--------|-------|
| LoctnH:Trt1 | -0.707 | -0.354 | -0.354 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 |
| LoctnA:Trt2 | -0.354 | -0.707 | -0.354 | 0.500 | 0.250 | 0.250 | 0.250 | 0.250 | 0.250 |
| LoctnB:Trt2 | -0.354 | -0.707 | -0.354 | 0.250 | 0.500 | 0.250 | 0.250 | 0.250 | 0.250 |
| LoctnC:Trt2 | -0.354 | -0.707 | -0.354 | 0.250 | 0.250 | 0.500 | 0.250 | 0.250 | 0.250 |
| LoctnD:Trt2 | -0.354 | -0.707 | -0.354 | 0.250 | 0.250 | 0.250 | 0.500 | 0.250 | 0.250 |
| LoctnE:Trt2 | -0.354 | -0.707 | -0.354 | 0.250 | 0.250 | 0.250 | 0.250 | 0.500 | 0.250 |
| LoctnF:Trt2 | -0.354 | -0.707 | -0.354 | 0.250 | 0.250 | 0.250 | 0.250 | 0.250 | 0.500 |
| LoctnG:Trt2 | -0.354 | -0.707 | -0.354 | 0.250 | 0.250 | 0.250 | 0.250 | 0.250 | 0.250 |
| LoctnH:Trt2 | -0.354 | -0.707 | -0.354 | 0.250 | 0.250 | 0.250 | 0.250 | 0.250 | 0.250 |
| LoctnA:Trt3 | -0.354 | -0.354 | -0.707 | 0.500 | 0.250 | 0.250 | 0.250 | 0.250 | 0.250 |
| LoctnB:Trt3 | -0.354 | -0.354 | -0.707 | 0.250 | 0.500 | 0.250 | 0.250 | 0.250 | 0.250 |
| LoctnC:Trt3 | -0.354 | -0.354 | -0.707 | 0.250 | 0.250 | 0.500 | 0.250 | 0.250 | 0.250 |
| LoctnD:Trt3 | -0.354 | -0.354 | -0.707 | 0.250 | 0.250 | 0.250 | 0.500 | 0.250 | 0.250 |
| LoctnE:Trt3 | -0.354 | -0.354 | -0.707 | 0.250 | 0.250 | 0.250 | 0.250 | 0.500 | 0.250 |
| LoctnF:Trt3 | -0.354 | -0.354 | -0.707 | 0.250 | 0.250 | 0.250 | 0.250 | 0.250 | 0.500 |
| LoctnG:Trt3 | -0.354 | -0.354 | -0.707 | 0.250 | 0.250 | 0.250 | 0.250 | 0.250 | 0.250 |
| LoctnH:Trt3 | -0.354 | -0.354 | -0.707 | 0.250 | 0.250 | 0.250 | 0.250 | 0.250 | 0.250 |
| LcG:T1 | LcH:T1 | LcA:T2 | LcB:T2 | LcC:T2 | LcD:T2 | LcE:T2 | LcF:T2 | LcG:T2 | |
| LocationA | | | | | | | | | |
| LocationB | | | | | | | | | |
| LocationC | | | | | | | | | |
| LocationD | | | | | | | | | |
| LocationE | | | | | | | | | |
| LocationF | | | | | | | | | |
| LocationG | | | | | | | | | |
| LocationH | | | | | | | | | |
| Trt1 | | | | | | | | | |
| Trt2 | | | | | | | | | |
| Trt3 | | | | | | | | | |
| LoctnA:Trt1 | | | | | | | | | |
| LoctnB:Trt1 | | | | | | | | | |
| LoctnC:Trt1 | | | | | | | | | |
| LoctnD:Trt1 | | | | | | | | | |
| LoctnE:Trt1 | | | | | | | | | |
| LoctnF:Trt1 | | | | | | | | | |
| LoctnG:Trt1 | | | | | | | | | |
| LoctnH:Trt1 | 0.500 | | | | | | | | |
| LoctnA:Trt2 | 0.250 | 0.250 | | | | | | | |
| LoctnB:Trt2 | 0.250 | 0.250 | 0.500 | | | | | | |
| LoctnC:Trt2 | 0.250 | 0.250 | 0.500 | 0.500 | | | | | |

| | | | | | | | | | |
|-------------|--------|--------|--------|--------|--------|--------|--------|--------|-------|
| LoctnD:Trt2 | 0.250 | 0.250 | 0.500 | 0.500 | 0.500 | | | | |
| LoctnE:Trt2 | 0.250 | 0.250 | 0.500 | 0.500 | 0.500 | 0.500 | | | |
| LoctnF:Trt2 | 0.250 | 0.250 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | | |
| LoctnG:Trt2 | 0.500 | 0.250 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | |
| LoctnH:Trt2 | 0.250 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 |
| LoctnA:Trt3 | 0.250 | 0.250 | 0.500 | 0.250 | 0.250 | 0.250 | 0.250 | 0.250 | 0.250 |
| LoctnB:Trt3 | 0.250 | 0.250 | 0.250 | 0.500 | 0.250 | 0.250 | 0.250 | 0.250 | 0.250 |
| LoctnC:Trt3 | 0.250 | 0.250 | 0.250 | 0.250 | 0.500 | 0.250 | 0.250 | 0.250 | 0.250 |
| LoctnD:Trt3 | 0.250 | 0.250 | 0.250 | 0.250 | 0.250 | 0.500 | 0.250 | 0.250 | 0.250 |
| LoctnE:Trt3 | 0.250 | 0.250 | 0.250 | 0.250 | 0.250 | 0.250 | 0.500 | 0.250 | 0.250 |
| LoctnF:Trt3 | 0.250 | 0.250 | 0.250 | 0.250 | 0.250 | 0.250 | 0.250 | 0.500 | 0.250 |
| LoctnG:Trt3 | 0.500 | 0.250 | 0.250 | 0.250 | 0.250 | 0.250 | 0.250 | 0.250 | 0.500 |
| LoctnH:Trt3 | 0.250 | 0.500 | 0.250 | 0.250 | 0.250 | 0.250 | 0.250 | 0.250 | 0.250 |
| | LcH:T2 | LcA:T3 | LcB:T3 | LcC:T3 | LcD:T3 | LcE:T3 | LcF:T3 | LcG:T3 | |
| LocationA | | | | | | | | | |
| LocationB | | | | | | | | | |
| LocationC | | | | | | | | | |
| LocationD | | | | | | | | | |
| LocationE | | | | | | | | | |
| LocationF | | | | | | | | | |
| LocationG | | | | | | | | | |
| LocationH | | | | | | | | | |
| Trt1 | | | | | | | | | |
| Trt2 | | | | | | | | | |
| Trt3 | | | | | | | | | |
| LoctnA:Trt1 | | | | | | | | | |
| LoctnB:Trt1 | | | | | | | | | |
| LoctnC:Trt1 | | | | | | | | | |
| LoctnD:Trt1 | | | | | | | | | |
| LoctnE:Trt1 | | | | | | | | | |
| LoctnF:Trt1 | | | | | | | | | |
| LoctnG:Trt1 | | | | | | | | | |
| LoctnH:Trt1 | | | | | | | | | |
| LoctnA:Trt2 | | | | | | | | | |
| LoctnB:Trt2 | | | | | | | | | |
| LoctnC:Trt2 | | | | | | | | | |
| LoctnD:Trt2 | | | | | | | | | |
| LoctnE:Trt2 | | | | | | | | | |
| LoctnF:Trt2 | | | | | | | | | |
| LoctnG:Trt2 | | | | | | | | | |

```

LoctnH:Trt2
LoctnA:Trt3 0.250
LoctnB:Trt3 0.250 0.500
LoctnC:Trt3 0.250 0.500 0.500
LoctnD:Trt3 0.250 0.500 0.500 0.500
LoctnE:Trt3 0.250 0.500 0.500 0.500 0.500
LoctnF:Trt3 0.250 0.500 0.500 0.500 0.500 0.500
LoctnG:Trt3 0.250 0.500 0.500 0.500 0.500 0.500 0.500
LoctnH:Trt3 0.500 0.500 0.500 0.500 0.500 0.500 0.500 0.500
> anova(fm1Mult)
Analysis of Variance Table
              Df Sum Sq Mean Sq
Location      8 6.9475  0.8684
Trt           3 1.2217  0.4072
Location:Trt 24 0.9966  0.0415
> (fm2Mult <- lmer(Adj ~ Location + Trt + (1 | Grp), Multilocation))
Linear mixed-effects model fit by REML
Formula: Adj ~ Location + Trt + (1 | Grp)
Data: Multilocation
AIC    BIC logLik MLdeviance REMLdeviance
20 54.87  3.001    -51.22      -6.001
Random effects:
Groups   Name             Variance Std.Dev.
Grp      (Intercept) 0.0050851 0.07131
Residual                    0.0367154 0.19161
number of obs: 108, groups: Grp, 27

Fixed effects:
              Estimate Std. Error t value
(Intercept)  2.53296    0.07599   33.33
LocationA    0.47818    0.09752    4.90
LocationB   -0.22443    0.09752   -2.30
LocationC    0.52712    0.09752    5.41
LocationD    0.02902    0.09752    0.30
LocationE    0.32537    0.09752    3.34
LocationF    0.73709    0.09752    7.56
LocationG    0.32098    0.09752    3.29
LocationH    0.80099    0.09752    8.21
Trt1         0.05834    0.05215    1.12
Trt2        -0.18802    0.05215   -3.61

```

```
Trt3          0.08379    0.05215    1.61
```

Correlation of Fixed Effects:

```
(Intr) LoctnA LoctnB LoctnC LoctnD LoctnE LoctnF LoctnG LoctnH
LocationA -0.642
LocationB -0.642  0.500
LocationC -0.642  0.500  0.500
LocationD -0.642  0.500  0.500  0.500
LocationE -0.642  0.500  0.500  0.500  0.500
LocationF -0.642  0.500  0.500  0.500  0.500  0.500
LocationG -0.642  0.500  0.500  0.500  0.500  0.500  0.500
LocationH -0.642  0.500  0.500  0.500  0.500  0.500  0.500  0.500
Trt1      -0.343  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000
Trt2      -0.343  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000
Trt3      -0.343  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000
      Trt1    Trt2
```

```
LocationA
LocationB
LocationC
LocationD
LocationE
LocationF
LocationG
LocationH
```

```
Trt1
Trt2      0.500
Trt3      0.500  0.500
```

```
> (fm3Mult <- lmer(Adj ~ Location + (1 | Grp), Multilocation))
```

Linear mixed-effects model fit by REML

Formula: Adj ~ Location + (1 | Grp)

Data: Multilocation

```
   AIC   BIC logLik MLdeviance REMLdeviance
29.82 56.64  -4.91    -22.17         9.82
```

Random effects:

```
Groups   Name             Variance Std.Dev.
Grp      (Intercept) 0.0016543 0.040673
Residual                0.0504389 0.224586
```

number of obs: 108, groups: Grp, 27

Fixed effects:

| | Estimate | Std. Error | t value |
|-------------|----------|------------|---------|
| (Intercept) | 2.52149 | 0.06895 | 36.57 |
| LocationA | 0.47818 | 0.09752 | 4.90 |
| LocationB | -0.22443 | 0.09752 | -2.30 |
| LocationC | 0.52712 | 0.09752 | 5.41 |
| LocationD | 0.02902 | 0.09752 | 0.30 |
| LocationE | 0.32537 | 0.09752 | 3.34 |
| LocationF | 0.73709 | 0.09752 | 7.56 |
| LocationG | 0.32098 | 0.09752 | 3.29 |
| LocationH | 0.80099 | 0.09752 | 8.21 |

Correlation of Fixed Effects:

| | (Intr) | LoctnA | LoctnB | LoctnC | LoctnD | LoctnE | LoctnF | LoctnG |
|-----------|--------|--------|--------|--------|--------|--------|--------|--------|
| LocationA | -0.707 | | | | | | | |
| LocationB | -0.707 | 0.500 | | | | | | |
| LocationC | -0.707 | 0.500 | 0.500 | | | | | |
| LocationD | -0.707 | 0.500 | 0.500 | 0.500 | | | | |
| LocationE | -0.707 | 0.500 | 0.500 | 0.500 | 0.500 | | | |
| LocationF | -0.707 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | | |
| LocationG | -0.707 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | |
| LocationH | -0.707 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 |

```
> (fm4Mult <- lmer(Adj ~ Trt + (1 | Grp), Multilocation))
```

Linear mixed-effects model fit by REML

Formula: Adj ~ Trt + (1 | Grp)

Data: Multilocation

| AIC | BIC | logLik | MLdeviance | REMLdeviance |
|-------|-------|--------|------------|--------------|
| 41.51 | 54.92 | -15.75 | 14.95 | 31.51 |

Random effects:

| Groups | Name | Variance | Std.Dev. |
|----------|-------------|----------|----------|
| Grp | (Intercept) | 0.110923 | 0.33305 |
| Residual | | 0.036715 | 0.19161 |

number of obs: 108, groups: Grp, 27

Fixed effects:

| | Estimate | Std. Error | t value |
|-------------|----------|------------|---------|
| (Intercept) | 2.86567 | 0.07395 | 38.75 |
| Trt1 | 0.05834 | 0.05215 | 1.12 |
| Trt2 | -0.18802 | 0.05215 | -3.61 |
| Trt3 | 0.08379 | 0.05215 | 1.61 |

```

Correlation of Fixed Effects:
      (Intr) Trt1    Trt2
Trt1 -0.353
Trt2 -0.353  0.500
Trt3 -0.353  0.500  0.500
> (fm5Mult <- lmer(Adj ~ 1 + (1 | Grp), Multilocation))
Linear mixed-effects model fit by REML
Formula: Adj ~ 1 + (1 | Grp)
Data: Multilocation
      AIC      BIC logLik MLdeviance REMLdeviance
51.33 56.69 -23.66      43.75      47.33
Random effects:
Groups   Name             Variance Std.Dev.
Grp      (Intercept) 0.107491 0.32786
Residual                    0.050439 0.22459
number of obs: 108, groups: Grp, 27

Fixed effects:
              Estimate Std. Error t value
(Intercept)  2.85419    0.06669   42.79
> anova(fm2Mult)
Analysis of Variance Table
              Df Sum Sq Mean Sq
Location     8  7.3768  0.9221
Trt          3  1.2217  0.4072
> (fm2MultR <- lmer(Adj ~ Trt + (Trt - 1 | Location) + (1 |
+   Block), Multilocation, control = list(msV = 1, niterEM = 200)))
  0      58.1951: 0.888889 0.888889 0.888889 0.888889 0.00000 0.00000 0.00
  1      24.0707:  1.03246 0.969626 0.994050 0.990267 -0.406756 -0.437447 -0.
  2      15.9680: 0.884835 0.815370  1.10073  1.41171 -0.341165 -0.401696 -0.
  3      15.4463: 0.977776 0.604681 0.969811  1.58336 0.0457936 -0.0495801 -1
  4      15.4463: 0.977776 0.604681 0.969811  1.58336 0.0457936 -0.0495801 -1
  5      15.4463: 0.977776 0.604681 0.969811  1.58336 0.0457936 -0.0495801 -1
Linear mixed-effects model fit by REML
Formula: Adj ~ Trt + (Trt - 1 | Location) + (1 | Block)
Data: Multilocation
      AIC      BIC logLik MLdeviance REMLdeviance
45.45 85.68 -7.723      2.553      15.45
Random effects:
Groups   Name             Variance Std.Dev.  Corr

```

```

Location Trt1      1.4556e-01 3.8152e-01
          Trt2      7.6956e-02 2.7741e-01 0.716
          Trt3      1.0610e-01 3.2574e-01 0.719 0.653
          Trt4      5.8897e-02 2.4269e-01 0.866 0.840 0.812
Block    (Intercept) 1.8599e-11 4.3126e-06
Residual              3.7197e-02 1.9287e-01
number of obs: 108, groups: Location, 9; Block, 3

```

Fixed effects:

| | Estimate | Std. Error | t value |
|-------------|----------|------------|---------|
| (Intercept) | 2.86567 | 0.08900 | 32.20 |
| Trt1 | 0.05834 | 0.08755 | 0.67 |
| Trt2 | -0.18802 | 0.07271 | -2.59 |
| Trt3 | 0.08379 | 0.08257 | 1.01 |

Correlation of Fixed Effects:

| | (Intr) | Trt1 | Trt2 |
|------|--------|-------|-------|
| Trt1 | 0.126 | | |
| Trt2 | -0.253 | 0.181 | |
| Trt3 | -0.107 | 0.249 | 0.177 |

I PBIB

```

> str(PBIB)
'data.frame':      60 obs. of  3 variables:
 $ response : num  2.4 2.5 2.6 2 2.7 2.8 2.4 2.7 2.6 2.8 ...
 $ Treatment: Factor w/ 15 levels "1","10","11",...: 7 15 1 5 11 13 14 1 2 1 ...
 $ Block    : Factor w/ 15 levels "1","10","11",...: 1 1 1 1 8 8 8 8 9 9 ...
- attr(*, "ginfo")=List of 7
 ..$ formula      :Class 'formula' length 3 response ~ Treatment | Block
 .. .. ..- attr(*, ".Environment")=length 24 <environment>
 ..$ order.groups: logi TRUE
 ..$ FUN          :function (x)
 ..$ outer        : NULL
 ..$ inner        : NULL
 ..$ labels       : list()
 ..$ units        : list()
> (fm1PBIB <- lmer(response ~ Treatment + (1 | Block), PBIB))
Linear mixed-effects model fit by REML
Formula: response ~ Treatment + (1 | Block)

```


Data: PBIB

| | | | | |
|-------|-------|--------|------------|--------------|
| AIC | BIC | logLik | MLdeviance | REMLdeviance |
| 83.98 | 117.5 | -25.99 | 22.83 | 51.98 |

Random effects:

| Groups | Name | Variance | Std.Dev. |
|----------|-------------|----------|----------|
| Block | (Intercept) | 0.046522 | 0.21569 |
| Residual | | 0.085559 | 0.29250 |

number of obs: 60, groups: Block, 15

Fixed effects:

| | Estimate | Std. Error | t value |
|-------------|-----------|------------|---------|
| (Intercept) | 2.891311 | 0.166413 | 17.374 |
| Treatment1 | -0.073789 | 0.222061 | -0.332 |
| Treatment10 | -0.400249 | 0.222061 | -1.802 |
| Treatment11 | 0.007388 | 0.222061 | 0.033 |
| Treatment12 | 0.161510 | 0.222061 | 0.727 |
| Treatment13 | -0.273542 | 0.222061 | -1.232 |
| Treatment14 | -0.400000 | 0.227200 | -1.761 |
| Treatment15 | -0.032078 | 0.222061 | -0.144 |
| Treatment2 | -0.485996 | 0.222061 | -2.189 |
| Treatment3 | -0.436368 | 0.222061 | -1.965 |
| Treatment4 | -0.107481 | 0.227200 | -0.473 |
| Treatment5 | -0.086413 | 0.222061 | -0.389 |
| Treatment6 | 0.019383 | 0.222061 | 0.087 |
| Treatment7 | -0.102326 | 0.222061 | -0.461 |
| Treatment8 | -0.109706 | 0.222061 | -0.494 |

Correlation of Fixed Effects:

| | (Intr) | Trtmn1 | Trtm10 | Trtm11 | Trtm12 | Trtm13 | Trtm14 | Trtm15 | Trtmn2 |
|-------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| Treatment1 | -0.667 | | | | | | | | |
| Treatment10 | -0.667 | 0.500 | | | | | | | |
| Treatment11 | -0.667 | 0.477 | 0.500 | | | | | | |
| Treatment12 | -0.667 | 0.500 | 0.500 | 0.500 | | | | | |
| Treatment13 | -0.667 | 0.500 | 0.500 | 0.500 | 0.500 | | | | |
| Treatment14 | -0.683 | 0.512 | 0.512 | 0.512 | 0.512 | 0.512 | | | |
| Treatment15 | -0.667 | 0.500 | 0.477 | 0.500 | 0.500 | 0.500 | 0.512 | | |
| Treatment2 | -0.667 | 0.500 | 0.500 | 0.500 | 0.477 | 0.500 | 0.512 | 0.500 | |
| Treatment3 | -0.667 | 0.500 | 0.500 | 0.500 | 0.500 | 0.477 | 0.512 | 0.500 | 0.500 |
| Treatment4 | -0.683 | 0.512 | 0.512 | 0.512 | 0.512 | 0.512 | 0.500 | 0.512 | 0.512 |
| Treatment5 | -0.667 | 0.500 | 0.477 | 0.500 | 0.500 | 0.500 | 0.512 | 0.477 | 0.500 |

| | | | | | | | | | |
|-------------|--------|--------|--------|--------|--------|-------|-------|-------|-------|
| Treatment6 | -0.667 | 0.477 | 0.500 | 0.477 | 0.500 | 0.500 | 0.512 | 0.500 | 0.500 |
| Treatment7 | -0.667 | 0.500 | 0.500 | 0.500 | 0.477 | 0.500 | 0.512 | 0.500 | 0.477 |
| Treatment8 | -0.667 | 0.500 | 0.500 | 0.500 | 0.500 | 0.477 | 0.512 | 0.500 | 0.500 |
| | Trtmn3 | Trtmn4 | Trtmn5 | Trtmn6 | Trtmn7 | | | | |
| Treatment1 | | | | | | | | | |
| Treatment10 | | | | | | | | | |
| Treatment11 | | | | | | | | | |
| Treatment12 | | | | | | | | | |
| Treatment13 | | | | | | | | | |
| Treatment14 | | | | | | | | | |
| Treatment15 | | | | | | | | | |
| Treatment2 | | | | | | | | | |
| Treatment3 | | | | | | | | | |
| Treatment4 | 0.512 | | | | | | | | |
| Treatment5 | 0.500 | 0.512 | | | | | | | |
| Treatment6 | 0.500 | 0.512 | 0.500 | | | | | | |
| Treatment7 | 0.500 | 0.512 | 0.500 | 0.500 | | | | | |
| Treatment8 | 0.477 | 0.512 | 0.500 | 0.500 | 0.500 | | | | |

J SIMS

```
> str(SIMS)
'data.frame':      3691 obs. of  3 variables:
 $ Pretot: num  29 38 31 31 29 23 23 33 30 32 ...
 $ Gain   : num   2  0  6  6  5  9  7  2  1  3 ...
 $ Class  : Factor w/ 190 levels "1","10","100",...: 1 1 1 1 1 1 1 1 1 1 ...
- attr(*, "ginfo")=List of 7
 ..$ formula      :Class 'formula' length 3 Gain ~ Pretot | Class
 .. ..- attr(*, ".Environment")=length 25 <environment>
 ..$ order.groups: logi TRUE
 ..$ FUN          :function (x)
 ..$ outer        : NULL
 ..$ inner        : NULL
 ..$ labels       :List of 2
 .. ..$ Pretot: chr "Sum of pre-test core item scores"
 .. ..$ Gain   : chr "Gain in mathematics achievement score"
 ..$ units       : list()
> (fm1SIMS <- lmer(Gain ~ Pretot + (Pretot | Class), SIMS))
Linear mixed-effects model fit by REML
Formula: Gain ~ Pretot + (Pretot | Class)
```

```

Data: SIMS
AIC    BIC logLik MLdeviance REMLdeviance
22391 22422 -11190      22373      22381
Random effects:
Groups   Name             Variance   Std.Dev. Corr
Class    (Intercept) 14.4894933 3.806507
          Pretot      0.0092028 0.095931 -0.641
Residual                22.2357557 4.715480
number of obs: 3691, groups: Class, 190

Fixed effects:
              Estimate Std. Error t value
(Intercept)   7.0596     0.3659   19.29
Pretot        -0.1860     0.0161  -11.56

Correlation of Fixed Effects:
      (Intr)
Pretot -0.760

```