

# Package ‘abclass’

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**Version** 0.5.1

**Description** Multi-category angle-based large-margin classifiers.

See Zhang and Liu (2014) <[doi:10.1093/biomet/asu017](https://doi.org/10.1093/biomet/asu017)> for details.

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abclass	<i>Multi-Category Angle-Based Classification</i>
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## Description

Multi-category angle-based large-margin classifiers with regularization by the elastic-net or group-wise penalty.

## Usage

```
abclass(
  x,
  y,
  loss = c("logistic", "boost", "hinge.boost", "lum"),
  penalty = c("glasso", "lasso"),
  weights = NULL,
  offset = NULL,
  intercept = TRUE,
  control = list(),
  ...
)
```

```
abclass.control(
  lum_a = 1,
  lum_c = 0,
  boost_umin = -5,
  alpha = 1,
  lambda = NULL,
  nlambdas = 50L,
  lambda_min_ratio = NULL,
  lambda_max_alpha_min = 0.01,
  penalty_factor = NULL,
  ncv_kappa = 0.1,
  gel_tau = 0.33,
  mellowmax_omega = 1,
  lower_limit = -Inf,
```

```

upper_limit = Inf,
epsilon = 1e-07,
maxit = 100000L,
standardize = TRUE,
varying_active_set = TRUE,
adjust_mm = FALSE,
save_call = FALSE,
verbose = 0L
)

```

## Arguments

<code>x</code>	A numeric matrix representing the design matrix. No missing values are allowed. The coefficient estimates for constant columns will be zero. Thus, one should set the argument <code>intercept</code> to <code>TRUE</code> to include an intercept term instead of adding an all-one column to <code>x</code> .
<code>y</code>	An integer vector, a character vector, or a factor vector representing the response label.
<code>loss</code>	A character value specifying the loss function. The available options are "logistic" for the logistic deviance loss, "boost" for the exponential loss approximating Boosting machines, "hinge.boost" for hybrid of SVM and AdaBoost machine, and "lum" for large-margin unified machines (LUM). See Liu, et al. (2011) for details.
<code>penalty</code>	A character vector specifying the name of the penalty.
<code>weights</code>	A numeric vector for nonnegative observation weights. Equal observation weights are used by default.
<code>offset</code>	An optional numeric matrix for offsets of the decision functions.
<code>intercept</code>	A logical value indicating if an intercept should be considered in the model. The default value is <code>TRUE</code> and the intercept is excluded from regularization.
<code>control</code>	A list of control parameters. See <code>abclass.control()</code> for details.
<code>...</code>	Other control parameters passed to <code>abclass.control()</code> .
<code>lum_a</code>	A positive number greater than one representing the parameter $a$ in LUM, which will be used only if <code>loss = "lum"</code> . The default value is <code>1.0</code> .
<code>lum_c</code>	A nonnegative number specifying the parameter $c$ in LUM, which will be used only if <code>loss = "hinge.boost"</code> or <code>loss = "lum"</code> . The default value is <code>1.0</code> .
<code>boost_umin</code>	A negative number for adjusting the boosting loss for the internal majorization procedure.
<code>alpha</code>	A numeric value in $[0,1]$ representing the mixing parameter $\alpha$ . The default value is <code>1.0</code> .
<code>lambda</code>	A numeric vector specifying the tuning parameter $\lambda$ . A data-driven $\lambda$ sequence will be generated and used according to specified <code>alpha</code> , <code>nlambda</code> and <code>lambda_min_ratio</code> if this argument is left as <code>NULL</code> by default. The specified $\lambda$ will be sorted in decreasing order internally and only the unique values will be kept.

<code>nlambda</code>	A positive integer specifying the length of the internally generated <i>lambda</i> sequence. This argument will be ignored if a valid <i>lambda</i> is specified. The default value is 50.
<code>lambda_min_ratio</code>	A positive number specifying the ratio of the smallest <i>lambda</i> parameter to the largest <i>lambda</i> parameter. The default value is set to $1e-4$ if the sample size is larger than the number of predictors, and $1e-2$ otherwise.
<code>lambda_max_alpha_min</code>	A positive number specifying the minimum denominator when the function determines the largest <i>lambda</i> . If the <i>lambda</i> is not specified, the largest <i>lambda</i> will be determined by the data and be the large enough <i>lambda</i> (that would result in all zero estimates for the covariates with positive penalty factors) divided by <code>max(alpha, lambda_max_alpha_min)</code> .
<code>penalty_factor</code>	A numerical vector with nonnegative values specifying the adaptive penalty factors for individual predictors (excluding intercept).
<code>ncv_kappa</code>	A positive number within $(0,1)$ specifying the ratio of reciprocal gamma parameter for group SCAD or group MCP. A close-to-zero <i>ncv_kappa</i> would give a solution close to lasso solution.
<code>gel_tau</code>	A positive parameter <i>tau</i> for group exponential lasso penalty.
<code>mellowmax_omega</code>	A positive parameter <i>omega</i> for Mellowmax penalty. It is experimental and subject to removal in future.
<code>lower_limit, upper_limit</code>	Numeric matrices representing the desired lower and upper limits for the coefficient estimates, respectively.
<code>epsilon</code>	A positive number specifying the relative tolerance that determines convergence.
<code>maxit</code>	A positive integer specifying the maximum number of iteration.
<code>standardize</code>	A logical value indicating if each column of the design matrix should be standardized internally to have mean zero and standard deviation equal to the sample size. The default value is TRUE. Notice that the coefficient estimates are always returned on the original scale.
<code>varying_active_set</code>	A logical value indicating if the active set should be updated after each cycle of coordinate-descent algorithm. The default value is TRUE for usually more efficient estimation procedure.
<code>adjust_mm</code>	An experimental logical value specifying if the estimation procedure should track loss function and adjust the MM lower bound if needed.
<code>save_call</code>	A logical value indicating if the function call of the model fitting should be saved. If TRUE, the function call will be saved in the returned object so that one can utilize <code>stats::update()</code> to update the argument specifications conveniently.
<code>verbose</code>	A nonnegative integer specifying if the estimation procedure is allowed to print out intermediate steps/results. The default value is 0 for silent estimation procedure.

**Value**

The function `abclass()` returns an object of class `abclass` representing a trained classifier; The function `abclass.control()` returns an object of class `abclass.control` representing a list of control parameters.

**References**

Zhang, C., & Liu, Y. (2014). Multicategory Angle-Based Large-Margin Classification. *Biometrika*, 101(3), 625–640.

Liu, Y., Zhang, H. H., & Wu, Y. (2011). Hard or soft classification? large-margin unified machines. *Journal of the American Statistical Association*, 106(493), 166–177.

**Examples**

```
library(abclass)
set.seed(123)

## toy examples for demonstration purpose
## reference: example 1 in Zhang and Liu (2014)
ntrain <- 100 # size of training set
ntest <- 1000 # size of testing set
p0 <- 2      # number of actual predictors
p1 <- 2      # number of random predictors
k <- 3       # number of categories

n <- ntrain + ntest; p <- p0 + p1
train_idx <- seq_len(ntrain)
y <- sample(k, size = n, replace = TRUE) # response
mu <- matrix(rnorm(p0 * k), nrow = k, ncol = p0) # mean vector
## normalize the mean vector so that they are distributed on the unit circle
mu <- mu / apply(mu, 1, function(a) sqrt(sum(a ^ 2)))
x0 <- t(sapply(y, function(i) rnorm(p0, mean = mu[i, ], sd = 0.25)))
x1 <- matrix(rnorm(p1 * n, sd = 0.3), nrow = n, ncol = p1)
x <- cbind(x0, x1)
train_x <- x[train_idx, ]
test_x <- x[- train_idx, ]
y <- factor(paste0("label_", y))
train_y <- y[train_idx]
test_y <- y[- train_idx]

## regularization through group lasso penalty
model <- abclass(
  x = train_x,
  y = train_y,
  loss = "logistic",
  penalty = "glasso"
)

pred <- predict(model, test_x, s = 5)
mean(test_y == pred) # accuracy
table(test_y, pred)
```

---

abclass_propscore	<i>Estimate Propensity Score by the Angle-Based Classifiers</i>
-------------------	---

---

## Description

A wrap function to estimate the propensity score by the multi-category angle-based large-margin classifiers.

## Usage

```
abclass_propscore(
  x,
  treatment,
  loss = c("logistic", "boost", "hinge.boost", "lum"),
  penalty = c("glasso", "gscad", "gmcp", "lasso", "scad", "mcp", "cmcp", "gel",
    "mellowmax", "mellowmcp"),
  weights = NULL,
  offset = NULL,
  intercept = TRUE,
  control = list(),
  tuning = c("et", "cv_1se", "cv_min"),
  ...
)
```

## Arguments

x	A numeric matrix representing the design matrix. No missing values are allowed. The coefficient estimates for constant columns will be zero. Thus, one should set the argument <code>intercept</code> to <code>TRUE</code> to include an intercept term instead of adding an all-one column to <code>x</code> .
treatment	The assigned treatments represented by a character, integer, numeric, or factor vector.
loss	A character value specifying the loss function. The available options are "logistic" for the logistic deviance loss, "boost" for the exponential loss approximating Boosting machines, "hinge.boost" for hybrid of SVM and AdaBoost machine, and "lum" for large-margin unified machines (LUM). See Liu, et al. (2011) for details.
penalty	A character vector specifying the name of the penalty.
weights	A numeric vector for nonnegative observation weights. Equal observation weights are used by default.
offset	An optional numeric matrix for offsets of the decision functions.
intercept	A logical value indicating if an intercept should be considered in the model. The default value is <code>TRUE</code> and the intercept is excluded from regularization.
control	A list of control parameters. See <code>abclass.control()</code> for details.

tuning	A character vector specifying the tuning method. This argument will be ignored if a single <code>lambda</code> is specified through <code>control</code> .
...	Other arguments passed to the corresponding methods.

coef.abclass

*Coefficient Estimates of A Trained Angle-Based Classifier***Description**

Extract coefficient estimates from an `abclass` object.

**Usage**

```
## S3 method for class 'abclass'
coef(object, selection = c("cv_1se", "cv_min", "all"), ...)
```

**Arguments**

object	An object of class <code>abclass</code> .
selection	An integer vector for the indices of solution path or a character value specifying how to select a particular set of coefficient estimates from the entire solution path. If the specified <code>abclass</code> object contains the cross-validation results, one may set <code>selection</code> to <code>"cv_min"</code> (or <code>"cv_1se"</code> ) for the estimates giving the smallest cross-validation error (or the set of estimates resulted from the largest <i>lambda</i> within one standard error of the smallest cross-validation error). The entire solution path will be returned in an array if <code>selection = "all"</code> or no cross-validation results are available in the specified <code>abclass</code> object.
...	Other arguments not used now.

**Value**

A matrix representing the coefficient estimates or an array representing all the selected solutions.

**Examples**

```
## see examples of `abclass()`.
```

---

coef.supclass

*Coefficient Estimates of A Trained Sup-Norm Classifier*


---

### Description

Extract coefficient estimates from an superclass object.

### Usage

```
## S3 method for class 'supclass'
coef(object, selection = c("cv_1se", "cv_min", "all"), ...)
```

### Arguments

object	An object of class superclass.
selection	An integer vector for the indices of solution or a character value specifying how to select a particular set of coefficient estimates from the entire solution path. If the specified superclass object contains the cross-validation results, one may set selection to "cv_min" (or "cv_1se") for the estimates giving the smallest cross-validation error (or the set of estimates resulted from the largest <i>lambda</i> within one standard error of the smallest cross-validation error). The entire solution path will be returned in an array if selection = "all" or no cross-validation results are available in the specified superclass object.
...	Other arguments not used now.

### Value

A matrix representing the coefficient estimates or an array representing all the selected solutions.

### Examples

```
## see examples of `supclass()`.
```

---

cv.abclass

*Tune Angle-Based Classifiers by Cross-Validation*


---

### Description

Tune the regularization parameter for an angle-based large-margin classifier by cross-validation.



**Usage**

```

cv.abclass(
  x,
  y,
  loss = c("logistic", "boost", "hinge.boost", "lum"),
  penalty = c("glasso", "lasso"),
  weights = NULL,
  offset = NULL,
  intercept = TRUE,
  control = list(),
  nfolds = 5L,
  stratified = TRUE,
  alignment = c("fraction", "lambda"),
  refit = FALSE,
  ...
)

```

**Arguments**

<code>x</code>	A numeric matrix representing the design matrix. No missing values are allowed. The coefficient estimates for constant columns will be zero. Thus, one should set the argument <code>intercept</code> to <code>TRUE</code> to include an intercept term instead of adding an all-one column to <code>x</code> .
<code>y</code>	An integer vector, a character vector, or a factor vector representing the response label.
<code>loss</code>	A character value specifying the loss function. The available options are "logistic" for the logistic deviance loss, "boost" for the exponential loss approximating Boosting machines, "hinge.boost" for hybrid of SVM and AdaBoost machine, and "lum" for large-margin unified machines (LUM). See Liu, et al. (2011) for details.
<code>penalty</code>	A character vector specifying the name of the penalty.
<code>weights</code>	A numeric vector for nonnegative observation weights. Equal observation weights are used by default.
<code>offset</code>	An optional numeric matrix for offsets of the decision functions.
<code>intercept</code>	A logical value indicating if an intercept should be considered in the model. The default value is <code>TRUE</code> and the intercept is excluded from regularization.
<code>control</code>	A list of control parameters. See <code>abclass.control()</code> for details.
<code>nfolds</code>	A positive integer specifying the number of folds for cross-validation. Five-folds cross-validation will be used by default. An error will be thrown out if the <code>nfolds</code> is specified to be less than 2.
<code>stratified</code>	A logical value indicating if the cross-validation procedure should be stratified by the response label. The default value is <code>TRUE</code> to ensure the same number of categories be used in validation and training.
<code>alignment</code>	A character vector specifying how to align the lambda sequence used in the main fit with the cross-validation fits. The available options are "fraction" for

	allowing cross-validation fits to have their own lambda sequences and "lambda" for using the same lambda sequence of the main fit. The option "lambda" will be applied if a meaningful lambda is specified. The default value is "fraction".
refit	A logical value indicating if a new classifier should be trained using the selected predictors or a named list that will be passed to <code>abclass.control()</code> to specify how the new classifier should be trained.
...	Other control parameters passed to <code>abclass.control()</code> .

### Value

An S3 object of class `cv.abclass` and `abclass`.

---

cv.moml	<i>MOML with Cross-Validation</i>
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---

### Description

Tune the regularization parameter for MOML by cross-validation.

### Usage

```
cv.moml(
  x,
  treatment,
  reward,
  propensity_score,
  loss = c("logistic", "boost", "hinge.boost", "lum"),
  penalty = c("glasso", "lasso"),
  weights = NULL,
  offset = NULL,
  intercept = TRUE,
  control = moml.control(),
  nfolds = 5L,
  stratified = TRUE,
  alignment = c("fraction", "lambda"),
  refit = FALSE,
  ...
)
```

### Arguments

x	A numeric matrix representing the design matrix. No missing values are allowed. The coefficient estimates for constant columns will be zero. Thus, one should set the argument <code>intercept</code> to <code>TRUE</code> to include an intercept term instead of adding an all-one column to <code>x</code> .
treatment	The assigned treatments represented by a character, integer, numeric, or factor vector.

reward	A numeric vector representing the rewards. It is assumed that a larger reward is more desirable.
propensity_score	A numeric vector taking values between 0 and 1 representing the propensity score.
loss	A character value specifying the loss function. The available options are "logistic" for the logistic deviance loss, "boost" for the exponential loss approximating Boosting machines, "hinge.boost" for hybrid of SVM and AdaBoost machine, and "lum" for largin-margin unified machines (LUM). See Liu, et al. (2011) for details.
penalty	A character vector specifying the name of the penalty.
weights	A numeric vector for nonnegative observation weights. Equal observation weights are used by default.
offset	An optional numeric matrix for offsets of the decision functions.
intercept	A logical value indicating if an intercept should be considered in the model. The default value is TRUE and the intercept is excluded from regularization.
control	A list of control parameters. See <code>abclass.control()</code> for details.
nfolds	A positive integer specifying the number of folds for cross-validation. Five-folds cross-validation will be used by default. An error will be thrown out if the nfolds is specified to be less than 2.
stratified	A logical value indicating if the cross-validation procedure should be stratified by the response label. The default value is TRUE to ensure the same number of categories be used in validation and training.
alignment	A character vector specifying how to align the lambda sequence used in the main fit with the cross-validation fits. The available options are "fraction" for allowing cross-validation fits to have their own lambda sequences and "lambda" for using the same lambda sequence of the main fit. The option "lambda" will be applied if a meaningful lambda is specified. The default value is "fraction".
refit	A logical value indicating if a new classifier should be trained using the selected predictors or a named list that will be passed to <code>abclass.control()</code> to specify how the new classifier should be trained.
...	Other arguments passed to the control function, which calls the <code>abclass.control()</code> internally.

cv.supclass

*Tune Sup-Norm Classifiers by Cross-Validation***Description**

Tune the regularization parameter lambda for a sup-norm classifier by cross-validation.

**Usage**

```
cv.supclass(
  x,
  y,
  model = c("logistic", "psvm", "svm"),
  penalty = c("lasso", "scad"),
  start = NULL,
  control = list(),
  nfolds = 5L,
  stratified = TRUE,
  ...
)
```

**Arguments**

<code>x</code>	A numeric matrix representing the design matrix. No missing values are allowed. The coefficient estimates for constant columns will be zero. Thus, one should set the argument <code>intercept</code> to <code>TRUE</code> to include an intercept term instead of adding an all-one column to <code>x</code> .
<code>y</code>	An integer vector, a character vector, or a factor vector representing the response label.
<code>model</code>	A character vector specifying the classification model. The available options are "logistic" for multi-nomial logistic regression model, "psvm" for proximal support vector machine (PSVM), "svm" for multi-category support vector machine.
<code>penalty</code>	A character vector specifying the penalty function for the sup-norms. The available options are "lasso" for sup-norm regularization proposed by Zhang et al. (2008) and "scad" for supSCAD regularization proposed by Li & Zhang (2021).
<code>start</code>	A numeric matrix representing the starting values for the quadratic approximation procedure behind the scene.
<code>control</code>	A list with named elements.
<code>nfolds</code>	A positive integer specifying the number of folds for cross-validation. Five-folds cross-validation will be used by default. An error will be thrown out if the <code>nfolds</code> is specified to be less than 2.
<code>stratified</code>	A logical value indicating if the cross-validation procedure should be stratified by the response label. The default value is <code>TRUE</code> to ensure the same number of categories be used in validation and training.
<code>...</code>	Other arguments passed to <code>supclass</code> .

**Value**

An S3 object of class `cv.supclass`.

et.abclass

*Tune Angle-Based Classifiers by ET-Lasso***Description**

Tune the regularization parameter for an angle-based large-margin classifier by the ET-Lasso method (Yang, et al., 2019).

**Usage**

```
et.abclass(
  x,
  y,
  loss = c("logistic", "boost", "hinge.boost", "lum"),
  penalty = c("glasso", "lasso"),
  weights = NULL,
  offset = NULL,
  intercept = TRUE,
  control = list(),
  nstages = 2L,
  nfolds = 0L,
  stratified = TRUE,
  alignment = c("fraction", "lambda"),
  refit = FALSE,
  ...
)
```

**Arguments**

x	A numeric matrix representing the design matrix. No missing values are allowed. The coefficient estimates for constant columns will be zero. Thus, one should set the argument <code>intercept</code> to <code>TRUE</code> to include an intercept term instead of adding an all-one column to <code>x</code> .
y	An integer vector, a character vector, or a factor vector representing the response label.
loss	A character value specifying the loss function. The available options are "logistic" for the logistic deviance loss, "boost" for the exponential loss approximating Boosting machines, "hinge.boost" for hybrid of SVM and AdaBoost machine, and "lum" for large-margin unified machines (LUM). See Liu, et al. (2011) for details.
penalty	A character vector specifying the name of the penalty.
weights	A numeric vector for nonnegative observation weights. Equal observation weights are used by default.
offset	An optional numeric matrix for offsets of the decision functions.
intercept	A logical value indicating if an intercept should be considered in the model. The default value is <code>TRUE</code> and the intercept is excluded from regularization.

control	A list of control parameters. See <code>abclass.control()</code> for details.
nstages	A positive integer specifying for the number of stages in the ET-Lasso procedure. By default, two rounds of tuning by random permutations will be performed as suggested in Yang, et al. (2019).
nfolds	A positive integer specifying the number of folds for cross-validation. Five-folds cross-validation will be used by default. An error will be thrown out if the <code>nfolds</code> is specified to be less than 2.
stratified	A logical value indicating if the cross-validation procedure should be stratified by the response label. The default value is TRUE to ensure the same number of categories be used in validation and training.
alignment	A character vector specifying how to align the lambda sequence used in the main fit with the cross-validation fits. The available options are "fraction" for allowing cross-validation fits to have their own lambda sequences and "lambda" for using the same lambda sequence of the main fit. The option "lambda" will be applied if a meaningful lambda is specified. The default value is "fraction".
refit	A logical value indicating if a new classifier should be trained using the selected predictors or a named list that will be passed to <code>abclass.control()</code> to specify how the new classifier should be trained.
...	Other control parameters passed to <code>abclass.control()</code> .

### Details

The ET-Lasso procedure is intended for tuning the lambda parameter solely. The arguments regarding cross-validation, `nfolds`, `stratified`, and `alignment`, allow one to estimate the prediction accuracy by cross-validation for the model estimates resulted from the ET-Lasso procedure, which can be helpful for one to choose other tuning parameters (e.g., alpha).

### Value

An S3 object of class `et.abclass` and `abclass`.

### References

Yang, S., Wen, J., Zhan, X., & Kifer, D. (2019). ET-Lasso: A new efficient tuning of lasso-type regularization for high-dimensional data. In *Proceedings of the 25th ACM SIGKDD International Conference on Knowledge Discovery & Data Mining* (pp. 607–616).

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et.moml

---

MOML with ET-Lasso

---

### Description

Tune the regularization parameter for MOML by the ET-Lasso method (Yang, et al., 2019).

**Usage**

```
et.moml(
  x,
  treatment,
  reward,
  propensity_score,
  loss = c("logistic", "boost", "hinge.boost", "lum"),
  penalty = c("glasso", "lasso"),
  weights = NULL,
  offset = NULL,
  intercept = TRUE,
  control = list(),
  nstages = 2,
  nfolds = 0L,
  stratified = TRUE,
  alignment = c("fraction", "lambda"),
  refit = FALSE,
  ...
)
```

**Arguments**

x	A numeric matrix representing the design matrix. No missing values are allowed. The coefficient estimates for constant columns will be zero. Thus, one should set the argument <code>intercept</code> to <code>TRUE</code> to include an intercept term instead of adding an all-one column to <code>x</code> .
treatment	The assigned treatments represented by a character, integer, numeric, or factor vector.
reward	A numeric vector representing the rewards. It is assumed that a larger reward is more desirable.
propensity_score	A numeric vector taking values between 0 and 1 representing the propensity score.
loss	A character value specifying the loss function. The available options are "logistic" for the logistic deviance loss, "boost" for the exponential loss approximating Boosting machines, "hinge.boost" for hybrid of SVM and AdaBoost machine, and "lum" for largin-margin unified machines (LUM). See Liu, et al. (2011) for details.
penalty	A character vector specifying the name of the penalty.
weights	A numeric vector for nonnegative observation weights. Equal observation weights are used by default.
offset	An optional numeric matrix for offsets of the decision functions.
intercept	A logical value indicating if an intercept should be considered in the model. The default value is <code>TRUE</code> and the intercept is excluded from regularization.
control	A list of control parameters. See <code>abclass.control()</code> for details.

nstages	A positive integer specifying for the number of stages in the ET-Lasso procedure. By default, two rounds of tuning by random permutations will be performed as suggested in Yang, et al. (2019).
nfolds	A positive integer specifying the number of folds for cross-validation. Five-folds cross-validation will be used by default. An error will be thrown out if the nfolds is specified to be less than 2.
stratified	A logical value indicating if the cross-validation procedure should be stratified by the response label. The default value is TRUE to ensure the same number of categories be used in validation and training.
alignment	A character vector specifying how to align the lambda sequence used in the main fit with the cross-validation fits. The available options are "fraction" for allowing cross-validation fits to have their own lambda sequences and "lambda" for using the same lambda sequence of the main fit. The option "lambda" will be applied if a meaningful lambda is specified. The default value is "fraction".
refit	A logical value indicating if a new classifier should be trained using the selected predictors or a named list that will be passed to <code>abclass.control()</code> to specify how the new classifier should be trained.
...	Other arguments passed to the control function, which calls the <code>abclass.control()</code> internally.

## References

Yang, S., Wen, J., Zhan, X., & Kifer, D. (2019). ET-Lasso: A new efficient tuning of lasso-type regularization for high-dimensional data. In *Proceedings of the 25th ACM SIGKDD International Conference on Knowledge Discovery & Data Mining* (pp. 607–616).

---

moml	<i>Multi-Category Outcome-Weighted Margin-Based Learning (MOML)</i>
------	---

---

## Description

Performs the outcome-weighted margin-based learning for multicategory treatments proposed by Zhang, et al. (2020).

## Usage

```
moml(
  x,
  treatment,
  reward,
  propensity_score,
  loss = c("logistic", "boost", "hinge.boost", "lum"),
  penalty = c("glasso", "lasso"),
  weights = NULL,
  offset = NULL,
  intercept = TRUE,
```



```

    control = moml.control(),
    ...
)

moml.control(...)

```

## Arguments

<code>x</code>	A numeric matrix representing the design matrix. No missing values are allowed. The coefficient estimates for constant columns will be zero. Thus, one should set the argument <code>intercept</code> to <code>TRUE</code> to include an intercept term instead of adding an all-one column to <code>x</code> .
<code>treatment</code>	The assigned treatments represented by a character, integer, numeric, or factor vector.
<code>reward</code>	A numeric vector representing the rewards. It is assumed that a larger reward is more desirable.
<code>propensity_score</code>	A numeric vector taking values between 0 and 1 representing the propensity score.
<code>loss</code>	A character value specifying the loss function. The available options are "logistic" for the logistic deviance loss, "boost" for the exponential loss approximating Boosting machines, "hinge.boost" for hybrid of SVM and AdaBoost machine, and "lum" for large-margin unified machines (LUM). See Liu, et al. (2011) for details.
<code>penalty</code>	A character vector specifying the name of the penalty.
<code>weights</code>	A numeric vector for nonnegative observation weights. Equal observation weights are used by default.
<code>offset</code>	An optional numeric matrix for offsets of the decision functions.
<code>intercept</code>	A logical value indicating if an intercept should be considered in the model. The default value is <code>TRUE</code> and the intercept is excluded from regularization.
<code>control</code>	A list of control parameters. See <code>abclass.control()</code> for details.
<code>...</code>	Other arguments passed to the control function, which calls the <code>abclass.control()</code> internally.

## References

Zhang, C., Chen, J., Fu, H., He, X., Zhao, Y., & Liu, Y. (2020). Multicategory outcome weighted margin-based learning for estimating individualized treatment rules. *Statistica Sinica*, 30, 1857–1879.

predict.abclass

*Prediction by A Trained Angle-Based Classifier***Description**

Predict class labels or estimate conditional probabilities for the specified new data.

**Usage**

```
## S3 method for class 'abclass'
predict(
  object,
  newx,
  type = c("class", "probability", "link"),
  selection = c("cv_1se", "cv_min", "all"),
  newoffset = NULL,
  ...
)
```

**Arguments**

object	An object of class abclass.
newx	A numeric matrix representing the design matrix for predictions.
type	A character value specifying the desired type of predictions. The available options are "class" for predicted labels, "probability" for class conditional probability estimates, and "link" for decision functions.
selection	An integer vector for the solution indices or a character value specifying how to select a particular set of coefficient estimates from the entire solution path for prediction. If the specified object contains the cross-validation results, one may set selection to "cv_min" (or "cv_1se") for using the estimates giving the smallest cross-validation error (or the set of estimates resulted from the largest <i>lambda</i> within one standard error of the smallest cross-validation error) or prediction. The prediction for the entire solution path will be returned in a list if selection = "all" or no cross-validation results are available in the specified object.
newoffset	An optional numeric matrix for the offsets.
...	Other arguments not used now.

**Value**

A vector representing the predictions or a list containing the predictions for each set of estimates along the solution path.

**Examples**

```
## see examples of `abclass()`.
```

---

predict.supclass	<i>Predictions from A Trained Sup-Norm Classifier</i>
------------------	---

---

## Description

Predict class labels or estimate conditional probabilities for the specified new data.

## Usage

```
## S3 method for class 'supclass'
predict(
  object,
  newx,
  type = c("class", "probability", "link"),
  selection = c("cv_1se", "cv_min", "all"),
  ...
)
```

## Arguments

object	An object of class abclass.
newx	A numeric matrix representing the design matrix for predictions.
type	A character value specifying the desired type of predictions. The available options are "class" for predicted labels, "probability" for class conditional probability estimates, and "link" for decision functions.
selection	An integer vector for the solution indices or a character value specifying how to select a particular set of coefficient estimates from the entire solution path for prediction. If the specified object contains the cross-validation results, one may set selection to "cv_min" (or "cv_1se") for using the estimates giving the smallest cross-validation error (or the set of estimates resulted from the largest <i>lambda</i> within one standard error of the smallest cross-validation error) or prediction. The prediction for the entire solution path will be returned in a list if selection = "all" or no cross-validation results are available in the specified object.
...	Other arguments not used now.

## Value

A vector representing the predictions or a list containing the predictions for each set of estimates.

## Examples

```
## see examples of `supclass()`.
```

supclass

*Multi-Category Classifiers with Sup-Norm Regularization***Description**

Experimental implementations of multi-category classifiers with sup-norm penalties proposed by Zhang, et al. (2008) and Li & Zhang (2021).

**Usage**

```
supclass(
  x,
  y,
  model = c("logistic", "psvm", "svm"),
  penalty = c("lasso", "scad"),
  start = NULL,
  control = list(),
  ...
)

supclass.control(
  lambda = 0.1,
  adaptive_weight = NULL,
  scad_a = 3.7,
  maxit = 50,
  epsilon = 1e-04,
  shrinkage = 1e-04,
  ridge_lambda = NA,
  warm_start = TRUE,
  standardize = TRUE,
  Rglpk = list(verbose = TRUE, tm_limit = 6e+05),
  ...
)
```

**Arguments**

x	A numeric matrix representing the design matrix. No missing values are allowed. The coefficient estimates for constant columns will be zero. Thus, one should set the argument <code>intercept</code> to <code>TRUE</code> to include an intercept term instead of adding an all-one column to <code>x</code> .
y	An integer vector, a character vector, or a factor vector representing the response label.
model	A character vector specifying the classification model. The available options are "logistic" for multi-nomial logistic regression model, "psvm" for proximal support vector machine (PSVM), "svm" for multi-category support vector machine.

penalty	A character vector specifying the penalty function for the sup-norms. The available options are "lasso" for sup-norm regularization proposed by Zhang et al. (2008) and "scad" for supSCAD regularization proposed by Li & Zhang (2021).
start	A numeric matrix representing the starting values for the quadratic approximation procedure behind the scene.
control	A list with named elements.
...	Optional control parameters passed to the <code>supclass.control()</code> .
lambda	A numeric vector specifying the tuning parameter <i>lambda</i> . The default value is 0.1. Users should tune this parameter for a better model fit. The specified lambda will be sorted in decreasing order internally and only the unique values will be kept.
adaptive_weight	A numeric vector or matrix representing the adaptive penalty weights. The default value is NULL for equal weights. Zhang, et al. (2008) proposed two ways to employ the adaptive weights. The first approach applies the weights to the sup-norm of coefficient estimates, while the second approach applies element-wise multiplication to the weights and coefficient estimates inside the sup-norms. The first or second approach will be applied if a numeric vector or matrix is specified, respectively. The adaptive weights are supported for lasso penalty only.
scad_a	A positive number specifying the tuning parameter <i>a</i> in the SCAD penalty.
maxit	A positive integer specifying the maximum number of iteration. The default value is 50 as suggested in Li & Zhang (2021).
epsilon	A positive number specifying the relative tolerance that determines convergence.
shrinkage	A nonnegative tolerance to shrink estimates with sup-norm close enough to zero (within the specified tolerance) to zeros. The default value is 1e-4.
ridge_lambda	The tuning parameter lambda of the ridge penalty used to set the starting values for multinomial logistic models.
warm_start	A logical value indicating if the estimates from last lambda should be used as the starting values for the next lambda. If FALSE, the user-specified starting values will be used instead.
standardize	A logical value indicating if a standardization procedure should be performed so that each column of the design matrix has mean zero and standardization
Rglpk	A named list that consists of control parameters passed to <code>Rglpk_solve_LP()</code> .

## Details

For the multinomial logistic model or the proximal SVM model, this function utilizes the function `quadprog::solve.QP()` to solve the equivalent quadratic problem. For the multi-class SVM, this function utilizes GNU Linear Programming Kit (GLPK) to solve the equivalent linear programming problem via the package **Rglpk**. It is recommended to use a recent version of **GLPK**.

## References

- Zhang, H. H., Liu, Y., Wu, Y., & Zhu, J. (2008). Variable selection for the multicategory SVM via adaptive sup-norm regularization. *Electronic Journal of Statistics*, 2, 149–167.
- Li, N., & Zhang, H. H. (2021). Sparse learning with non-convex penalty in multi-classification. *Journal of Data Science*, 19(1), 56–74.

## Examples

```
library(abclass)

if (requireNamespace("quadprog", quietly = TRUE)) {
  ## toy examples for demonstration purpose
  ## reference: example 1 in Zhang and Liu (2014)
  set.seed(123)
  ntrain <- 100 # size of training set
  ntest <- 1000 # size of testing set
  p0 <- 2      # number of actual predictors
  p1 <- 2      # number of random predictors
  k <- 3       # number of categories

  n <- ntrain + ntest; p <- p0 + p1
  train_idx <- seq_len(ntrain)
  y <- sample(k, size = n, replace = TRUE) # response
  mu <- matrix(rnorm(p0 * k), nrow = k, ncol = p0) # mean vector
  ## normalize the mean vector so that they are distributed on the unit circle
  mu <- mu / apply(mu, 1, function(a) sqrt(sum(a ^ 2)))
  x0 <- t(sapply(y, function(i) rnorm(p0, mean = mu[i, ], sd = 0.25)))
  x1 <- matrix(rnorm(p1 * n, sd = 0.3), nrow = n, ncol = p1)
  x <- cbind(x0, x1)
  train_x <- x[train_idx, ]
  test_x <- x[- train_idx, ]
  y <- factor(paste0("label_", y))
  train_y <- y[train_idx]
  test_y <- y[- train_idx]

  ## regularization with the supnorm lasso penalty
  options("mc.cores" = 1)

  model <- supclass(train_x, train_y, model = "psvm", penalty = "lasso")
  pred <- predict(model, test_x)
  table(test_y, pred)
  mean(test_y == pred) # accuracy
}
```

---

vertex

*Simplex Vertices for The Angle-Based Classification*


---

## Description

Simplex Vertices for The Angle-Based Classification

## Usage

```
vertex(k)
```

**Arguments**

k                      Number of classes, a positive integer that is greater than one.

**Value**

A  $(k-1)$  by  $k$  matrix that consists of vertices in columns.

**References**

Lange, K., & Tong Wu, Tong (2008). An MM algorithm for multicategory vertex discriminant analysis. *Journal of Computational and Graphical Statistics*, 17(3), 527–544.

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